# V. PLANETARY NEBULAE CONNECTION: EVOLUTION TO WHITE DWARFS



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## **EVOLUTIONARY TRACKS**

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## Introduction

It is now accepted without any doubts that the central stars of Planetary Nebulae (CPN) are rapidly evolving objects in the transition from the asymptotic giant branch (AGB) to the white-dwarf regime. After the pioneering study of Paczyński (1971) it has been demonstrated by Schönberner (1981) that Paczyński's calculations are too crude for the understanding of post-AGB evolution because the latter depends very sensitively on the detailed internal stellar structure, i.e. on the past AGB evolution. More precisely, the evolution of an AGB remnant is a function of the thermal-pulse cycle phase  $\phi$  during which this remnant has been created by the planetary-nebula (PN) formation process. This has been shown by Schönberner (1979, 1983) and later fully been explored by Iben (1984).

In short, two major modes of post-AGB evolution towards the white-dwarf stage are possible, according to the two main modes of thermally pulsing AGB stars: the hydrogen-burning or helium-burning configuration. If, for instance, the PN formation, i. e. the removal of the stellar envelope by mass loss, occurs during the luminosity peak that immediately follows the helium shell instability, the remnant leaves the AGB while still only burning helium (Härm and Schwarzschild, 1975). On the other hand, PN formation may also occur during the quiescent hydrogenburning phase, and the remnant then continues to burn hydrogen on its way to becoming a white dwarf.

Details and timing of the PN formation with respect to the thermal-pulse cycle phase are not known because of our poor knowledge of the mass-loss processes on the AGB. The high sensitivity of the post-AGB evolutionary tracks to the thermalpulse phase  $\phi$  allows, however, a distinction to be made between the helium-burning  $(\phi = 0)$  and hydrogen-burning mode  $(\phi \ge 0.3)$  by observations. An illustration is given in Fig. 1 where the temporal variations of different luminosity contributions of a 0.63  $M_{\odot}$  remnant for initial phases  $\phi = 0$  and 0.85 resp. are shown (Blöcker, priv. comm.). The time is counted from a position close to the AGB (see below for more details). For hydrogen burning, the surface luminosity, L, remains constant ('plateau') until it drops rapidly by more than a factor of ten within a very short time span. Hydrogen burning becomes unimportant, and the further evolution is mainly controlled by the gravo-thermal energy release  $L_{\rm g}$ . A helium-burning remnant behaves differently: helium burning decays only slowly, and the changeover to  $L_{\rm g}$  occurs on a larger timescale.

Observational evidences in favour of hydrogen-burning post-AGB models have already been collected by Schönberner (1989) and shall not be discussed again. I will only mention new results from PN progenitors, i.e. from cool post-AGB stars with detached, somewhat diluted dusty shells (v.d. Veen et al., 1989). The observed  $T_{\rm eff}$ -post AGB age distribution is in excellent agreement with the predictions of hydrogen-burning post-AGB models with masses between 0.55 and  $0.65 M_{\odot}$  (cf. also Schönberner, 1990).

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Fig. 1. Different luminosity contributions for a hydrogen-burning (left) and helium-burning remnant (right) of  $0.63M_{\odot}$  vs. post-AGB ages. Thick lines: surface luminosities L.

It must, however, be acknowledged that a substantial fraction ( $\approx 10...30\%$ ) of the observed central-star population appears to have virtually hydrogen-free surfaces. Typical members belonging to this subclass are the WC, O VI and PG 1159 nuclei. Their origin is still quite uncertain but may be connected to a very 'late' helium shell flash that is accompanied by mixing and non-equilibrium burning (Iben et al., 1983). The reader is referred to Schönberner and Blöcker (1992) for a recent account of the problems in explaining hydrogen-free (i.e. helium-burning) CPN. We will consider here only hydrogen-burning central stars.

#### Mass loss and evolution along the AGB

The structure of an AGB star is characterized by a very compact hydrogenexhausted core with mass  $M_{\rm H}$  and a very dilute, fully convective envelope with mass  $M_{\rm e}$  containing the unprocessed matter. The hydrogen-burning shell is the interface between the core and the envelope, and the helium-burning shell sits further inside. The inert, helium-exhausted inner part of the stellar core consists of carbon and oxygen and is electron-degenerated. The stellar radius is about  $10^4$  times larger than that of the core.

The luminosity of a giant star at the Hayashi limit depends practically only on the mass of its core (Paczyński, 1970; Kippenhahn, 1981). This statement holds, in general, also for stars on the AGB since it is mainly hydrogen burning that determines the overall course of evolution, i.e. virtually  $L = L_{\rm H}$  for more than 80% of the evolutionary time. This luminosity,  $L_{\rm H}$ , in turn determines the growth rate,  $\dot{M}_{\rm H}$ , of the core according to

$$M_{\rm H} = L_{\rm H} / X E_{\rm H},$$

where  $E_{\rm H} (= 6.3 \cdot 10^{18} \,{\rm erg/g})$  is the energy released per gram of hydrogen and X the hydrogen mass fraction of the stellar envelope. For a typical core mass  $M_{\rm H}$  of  $0.6M_{\odot}$ ,  $\dot{M}_{\rm H}$  is about  $10^{-7}M_{\odot}/{\rm yr}$ . The core mass-luminosity relationship does not hold for massive stars where envelope convection cuts into the hydrogen-burning shell. The surface luminosity is substantially larger in these cases and depends also on the envelope mass. For more details see Blöcker and Schönberner (1991). Close to the tip of the AGB where the envelope mass is already small (<  $0.1M_{\odot}$ ), the core mass-luminosity relation is always valid.

The stellar core can be thought of as being a very hot (pre) white dwarf which accretes nuclearly processed matter from the envelope. It grows by mass at the expense of the envelope at a rate given above while its radius shrinks. The core evolves independently of the envelope as long as the latter contains sufficient mass as to keep up the burning temperature in the hydrogen shell source by contraction. This also means that the evolution is independent of mass loss as long as thermal equilibrium is maintained. The evolutionary track of an AGB star in the HR diagram is entirely due to the envelope's response to the (masswise) growing core: expansion of the outer parts along the Hayashi-limit in order to accommodate the increasing luminosity, but overall contraction at nearly constant luminosity when the envelope mass drops below  $\approx 0.05 M_{\odot}$ . If hydrogen burning cannot be sustained anymore ( $M_e < 10^{-4} M_{\odot}$ ), the luminosity drops and the envelope shrinks finally to white-dwarf dimensions (see also discussion in the next Section).

While the nuclear evolution of AGB stars is controlled by the core  $M_{\rm H}$ , the total lifetime on the AGB is determined by mass loss from the surface of the star. This is a consequence of the observed stellar winds with mass-loss rates up to  $\dot{M}_{\rm w} \approx 10^{-4} M_{\odot}/{\rm yr}$  for the most luminous AGB stars (e.g. Knapp, 1985). Even if many stars may not reach such large rates, it is clear that always  $\dot{M}_{\rm w} \gg \dot{M}_{\rm H}$  on the upper AGB (at 50000  $L_{\odot}$ ,  $\dot{M}_{\rm H} \simeq 7 \cdot 10^{-7} M_{\odot}/{\rm yr}$  only).

Defining an evolutionary timescale by

$$-\frac{M_{\rm e}}{\dot{M}_{\rm e}} = \frac{M_{\rm e}}{\dot{M}_{\rm H} + \dot{M}_{\rm w}} \qquad {\rm with} \ \dot{M}_{\rm H} > 0, \ \dot{M}_{\rm w} > 0, \ \dot{M}_{\rm e} < 0 \ ,$$

we immediately conclude that stellar evolution is determined by the wind term,  $\dot{M}_{\rm w}$ , on and in the vicinity of the AGB. Thus, the lifetime at the tip of the AGB is very short (planetary-nebula formation!), and the following post-AGB evolution is completely determined by the internal stellar structure at the AGB tip, which in turn is a function of the thermal-pulse cycle phase  $\phi$  (cf. Iben 1984, also Schönberner 1979, 1983, or previous Section).

A certain mass-loss law has to be specified in evolutionary calculations, with the constraint that the total amount of mass lost from the star, i.e.  $\int \dot{M}_{agb} dt$ , must be consistent with a reasonable initial-final mass relation. A simple Reimers law (Reimers, 1975) will not work since it predicts a too small integrated mass loss which is not consistent with the rather flat initial-final mass relationship of Fig. 2 (see also Weidemann, 1992). Instead, Blöcker and Schönberner (1990) employed a formula for  $\dot{M}_{\rm agb}$  adapted from Bowen's (1988) calculations of pulsating Mira atmospheres. In terms of the Reimers rate,  $\dot{M}_{\rm R}$ , they got  $\dot{M}_{\rm agb}/\dot{M}_{\rm R} \sim L^{2.7}$  which yields  $10^{-3}$  to  $10^{-4} M_{\odot}/{\rm yr}$  at the tip of the AGB. The resulting combinations  $(M_{\rm i}, M_{\rm f}) = 3M_{\odot}, 0.605M_{\odot}$  and  $5M_{\odot}, 0.836M_{\odot}$  resp., are in fair agreement with the empirical galactic initial-final mass relation of Weidemann (1987), as is illustrated in Fig. 2. There are also the sequences of Wood and Faulkner (1986) plotted, and one realizes immediately that only their combination  $(M_{\rm i}, M_{\rm f}) = 2M_{\odot}, 0.6M_{\odot}$  is consistent with the initial-final mass relation! Their more massive remnants have been computed from too small initial masses, which has consequences for the late central-star evolution (next Section).



Fig. 2. Empirical initial-final mass relationship from Weidemann (1987) with different evolutionary sequences as explained in the text.

## Evolution through the central-star region

The remnant's transition towards the hotter parts of the HR diagram is also influenced by mass loss. The situation is, however, worse than in the case of the AGB evolution because neither reliable observations nor a theory exists. Furthermore, we do not exactly know where (i.e. at which effective temperature) the strong AGB mass loss ceases. It is normally assumed that this happens when  $M_e \approx 10^{-3} M_{\odot}$ , corresponding to an effective temperature between  $5000 \dots 6000$  K (Schönberner, 1983; Wood and Faulkner 1986). For the post-AGB evolution mass loss has either been neglected (Wood and Faulkner 1986) or set equal to the Reimers rate (Schönberner,1979,1983). Since the latter predicts  $\dot{M}_w = \dot{M}_R \sim T_{\rm eff}^{-2}$ , such a wind term is only important for  $T_{\rm eff} < 10000$  K.

A more physical approach has recently been described by Blöcker and Schönberner (1990): the end of heavy AGB mass loss is coupled to the pulsational properties of the models, because the periods of radial pulsations decrease as the effective temperature becomes larger when the model is slowly departing from the Hayashi line. The pulsational periods are calculated from the stellar parameters according to Ostlie and Cox (1986). The mass-loss rate is reduced from its AGB value to the Reimers rate while the period (fundamental mode) changes from 100 to 50 d. A period of 50 d corresponds to  $T_{\rm eff} \approx 6000 \dots 6500$  K, depending on the remnant mass, and thus gives very similar zero points for the post-AGB evolution as used by Schönberner (1983). The Reimers rate is kept until it falls below the rate predicted by radiatively driven winds (Pauldrach et al., 1988) which is then used for central stars. This rate scales like  $\dot{M}_{\rm w} = \dot{M}_{\rm P} \sim L^{1.9}$ , roughly independent of effective temperature.  $\dot{M}_{\rm P}$  is well below the burning rate,  $\dot{M}_{\rm H}$ , and does not afflict the evolutionary speed of central stars (cf. Perinotto, 1989).

The two post-AGB models with 0.605 and  $0.836 M_{\odot}$ , resp., calculated by Blöcker and Schönberner (1990), behave on the 'plateau' as expected: the  $0.836 M_{\odot}$  model heats up about ten times faster than the  $0.605 M_{\odot}$  model. For the final dimming on the 'cooling' track, however, the results are different: at  $L \approx 10^{2.1} L_{\odot}$  the massive remnant is *twice* as old as the lighter one (cf. also Fig.3 in the next section). This result is in variance with the computations of Wood and Faulkner (1986) which predict a steadily increasing fading speed with increasing remnant mass.

An explanation is possible with Fig.1 (left part). When hydrogen-burning is extinct,  $L \simeq L_{\rm g}$ . Then the temporal evolution of the photon luminosity depends mainly on the thermo-mechanical structure of the core, modified somewhat by the neutrino losses. The structure of the core depends, on the other hand, on its evolutionary history: With increasing age (or thermal-pulse number) the core becomes smaller and denser, thereby steadily reducing its available potential-energy reservoir on its way to a fully electron degenerate configuration. We expect, in general, an increasing fading speed with increasing core age (core mass) or thermal-pulse number. Now more massive stars start their AGB evolution with already quite massive cores which are less condensed and substantially hotter than old cores of the same mass evolved from stars with smaller initial mass. These young and massive cores have a large reservoir of potential and thermal energy which enables them to fade with a considerably slower pace. With increasing age on the AGB such a massive core contracts and cools (neutrino losses!) until it converges to the limiting configuration for which heating by compression and cooling by neutrinos is nearly balanced. Only central stars which possess cores with such a 'converged' structure are expected to fade independently of their past evolution. For  $M_{\rm i} = 5 M_{\odot}$  about 40 thermal pulses are necessary, leading to  $M_{\rm H} \simeq 0.89 \, M_{\odot}$  (Blöcker, priv. comm.). It is questionable whether mass loss does allow such a long lifetime ( $\approx 2.5 \cdot 10^5 \text{yr}$ ) at very high luminosities  $(M_{\rm bol} < -6)$ .

The above discussion of fading properties did not consider the influence of the thermal-pulse cycle phase  $\phi$ . Indeed, over a complete pulse cycle  $L_{\rm g}$  does not remain constant. Large amounts of heat are converted into potential energy during the helium-shell instability by expanding the outer core regions. These regions contract after the shell flash and give rise to a slowly decreasing release of gravo-thermal energy. The core is smallest immediately before the onset of the next helium shell flash. This is the reason why Wood and Faulkner (1986) found a slight increase of the fading speed with  $\phi$ .

The different fading speeds of three selected AGB remnants are illustrated in Blöcker and Schönberner (poster, these proceedings). The tracks of the two remnants with the relatively young cores show clearly the importance of further contraction on the upper part of the 'cooling' track. The plateau evolution of the remnants with equal masses but different core ages, i.e. different initial masses, is virtually identical. This behavior corresponds with the earlier discussion above, since the evolution along the 'plateau' is controlled by the actual values of envelope mass, nuclear burning and mass loss. The influence of the earlier evolutionary history is only small. Note that the  $0.836M_{\odot}$  remnant with the older and smaller core is slightly more luminous.

#### Comparison with observations

For comparison with observations we need a larger set of theoretical tracks to cover the expected mass spectrum of CPN. We thus combined the results of Blöcker and Schönberner (1990) with the earlier calculations of Schönberner (1983) and determined post-AGB isochrones (Fig. 3). These isochrones bend sharply downwards because of the fast luminosity drop when hydrogen burning extinguishes, and then upwards because of the slow fading of our massive AGB remnant. These isochrones suggest that the least luminous central stars should have about  $0.65 M_{\odot}$ !



Fig. 3. HR diagram with hydrogen-burning post-AGB tracks and isochrones.

The 'plateau' luminosities have very successfully been determined by observing and analysing Magellanic Cloud PN (cf. Walton et al., 1991). Upon comparison with the tracks they found a very narrow range of CPN masses,  $0.58...0.64M_{\odot}$ , with a mean mass of  $M = 0.59 \pm 0.2 M_{\odot}$ . In respect with the isochrones depicted in Fig. 3 it is, however, more interesting to study the late CPN evolution. For this purpose we used the fact that many PN with hot, intrinsically faint central stars appear to be optically thick even in the hydrogen Lyman continuum, making a temperature determination by the Zanstra method possible. Also model PN coupled to hydrogen-burning central-star models show that recombination caused by the rapid luminosity drop leads temporarily to optically thick nebular shells (Schönberner and Tylenda, 1990; Marten and Schönberner, 1991). The ionized masses are a few tenth of a solar mass, and the well-known Shklovsky method for distance determinations is in these cases applicable! We thus selected all clear-cut cases of optically thick galactic PN from the lists of Jacoby and Kaler (1989) and Kaler and Jacoby (1989), resulting in a sample of 29 very hot central stars (out of 82 listed objects). Their loci in the HR diagram are estimated from the Zanstra temperatures  $(T_{\rm HI} \simeq T_{\rm HeII})$ and Shklovsky distances  $(M_{\rm ion} = 0.2 M_{\odot})$  and compared to the theoretical tracks in Fig. 4. The agreement with our calculations is striking: practically all objects accumulate where the fading speed of the models decreases, with the least luminous objects having indeed only modest masses of, say, a little bit above  $0.63 M_{\odot}$ . The hottest (and most massive) nuclei have comparatively larger luminosities, just as our calculations predict. A few objects appear to be rather young, with still high luminosities. They are likely also more massive CPN which evolve so quickly along the 'plateau' that the usual optically-thin PN phase does not occur.

The ionized nebular mass,  $M_{\rm ion}$ , for the oldest objects plotted in Fig.4 may have been underestimated because of accretion of matter from the old AGB wind (cf. Marten and Schönberner (1991), Fig. 12). Thus distances to hot, low-luminous CPN are very likely *underestimated*, contrary to the general belief (cf. Kaler and Jacoby, 1989). Increasing the distances to these objects would not change our conclusion.

In conclusion we can summarize that the late evolution of more massive central stars depends on their history and is not a unique function of their core mass as normally assumed in the past. Moreover, the investigation of a larger sample of low-luminosity CPN indicates that relatively massive AGB stars experience only few thermal pulses before PN ejection, and that the resulting remnants have young cores which fade rather slowly along the upper part of the 'cooling' track. It would be very important for the future to concentrate on the hottest central stars and to improve their temperature and luminosity determinations.

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Fig. 4. HR diagram of hot central stars with optically thick planetaries.

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