CORRIGENDUM
to the paper

ANNIHILATORS AND THE CS-CONDITION

by W. K. NICHOLSON and M. F. YOUSIF

Proposition 3.1 of the paper Annihilators and the CS-condition, *Glasgow Math. J.* **40** (1998), 213–222, is incorrect as stated, and consequently the note added in proof is incorrect. Hence the question of Faith and Menal whether every strongly right Johns ring is quasi-Frobenius remains open. The problem is that the assumption that the left socle S_l and the right socle S_r are equal is not established. All we know is that $S_l \subseteq S_r = r(J) = l(J)$ by [**8**, Lemma 2.2]. We can prove the following result.

THEOREM. *The following conditions are equivalent for ring R for which the matrix ring $M_2(R)$ is right Johns.*

(a) *R is right mininjective.*
(b) *kR simple, $k \in R$, implies that Rk is simple.*
(c) *R is semilocal.*
(d) *$S_r \subseteq S_l$ (so $S_r = S_l$).*
(e) *R is left Kasch.*
(f) *R is quasi-Frobenius.*

Proof. If $M_2(R)$ is right Johns, it is not difficult to see that R is right Johns. Moreover, R is left 2-injective by [**15**, Theorem 4.2] because $M_2(R)$ is left P-injective.

(a) $\Rightarrow$ (b). This is by [**16**, Theorem. 1.14].

(b) $\Rightarrow$ (c). Since R is right noetherian, $S_r = k_1R \oplus \cdots \oplus k_nR$ where the k_iR are simple. By [**8**, Lemma 2.2] we have $J = l(S_r) = l(k_l) \cap \cdots \cap l(k_n)$, and we are done because each Rk_2 is simple by (b).

(c) $\Rightarrow$ (d). If R is semilocal, then $r(J) = S_l$ and so (d) follows because $S_r = r(J)$ by [**8**, Lemma 2.2].

(d) $\Rightarrow$ (a). R is left mininjective because it is left P-injective (being right Johns). Hence, if Rk is a minimal left ideal of R, then kR is a minimal right ideal of R by [**16**, Theorem 1.14]. This means $J \subseteq r(k)$, so $Rk \subseteq lr(k) \subseteq l(J) = S_r$, by [**8**, Lemma 2.2]. Hence $lr(k) \subseteq S_l$ by (d), so $lr(k)$ is a semisimple left R-module containing Rk. Thus it suffices to show that $Rk \subseteq^{ess} lr(k)$. Suppose that $0 \neq y \in lr(k)$. Observe first that $r(k) \subseteq r(y) \neq R$, whence $r(k) = r(y)$ and $lr(k) = lr(y)$. Now suppose to the contrary that $Rk \cap Ry = 0$. Then $R = r(Rk \cap Ry) - r(k) + r(y)$ because R is left 2-injective. This implies that $0 = l/r(k) + r(y)] = lr(k) \cap lr(y) = lr(k)$, a contradiction.

(c) $\Rightarrow$ (e). R is right mininjective because (c) $\Rightarrow$ (a), and it is left mininjective by hypothesis. As R is right Kasch, let $k_1R, \cdots, k_nR, k_i \in R$, be a complete set of representatives of the simple right modules. Then [**16**, Theorem 1.14) shows that each Rk_i is simple (by right mininjectivity), and that $Rk_i \cong Rk_j$ implies $k_iR \cong k_jR$ (by left mininjectivity), whence $i=j$. Since R is semilocal, $Rk_1, \ldots, Rk_n$ are a complete set of representatives of the simple left modules, proving (e).

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(e) $\Rightarrow$ (a). Because R is left 2-injective, it is right P-injective by [**15**, Lemma 2.2], and (a) follows.

(c) $\Rightarrow$ (f). Clearly (f) $\Rightarrow$ (c). Conversely, as R is right noetherian and J is nilpotent (by [**8**, Lemma 2.2]), R is right artinian by Hopkins' Theorem. But R is right mininjective because (c) $\Rightarrow$ (a), and R is left mininjective by hypothesis, so that R is quasi-Frobenius by [**16**, Corollary 4.8]. □

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