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Convection during the formation of gaseous giants and stars

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Abstract. Convection theories for star and planet formation studies have to be (1) simple, to allow a self-consistent solution with other relevant processes, (2) time-dependent, because convection often starts in collapse-flows, and (3) robust, i.e. physically well-behaved under a wide range of conditions ranging from the quiet protoplanetary nebula to supercritical protostellar accretion-shocks with Mach-numbers of a few hundred. I describe how the equations of radiation fluid-dynamics can be augmented by a one-equation convection model in order to construct a system of equations that contains the Sun, brown dwarfs and planets as well as their nearly isothermal parent-clouds. The system of equations is calibrated to the Sun and tested by the solar convection zone and the pulsations of RR-Lyrae stars. I discuss the following applications: (1) star formation as the collapse of Bonnor-Ebert spheres of masses ranging from the stellar domain to the brown dwarf region, (2) the approach to the main sequence, (3) companion mass determinations for direct imaging searches for exoplanets, with GQ Lupi as an example, and (4) the formation of Pegasi-planets, and the "large core" exoplanet, HD 149 026, in particular.

Keywords. Convection, planets and satellites: formation, solar system: formation, stars: formation, stars: pre–main-sequence, (stars:) planetary systems: formation

1. Introduction: The youngest celestial bodies are convective

Young celestial bodies are cool, large and more luminous than in the more mature, long-lasting phases of their life. Because they have not yet been fully compressed, degeneracy plays a rather minor role and so does heat conduction. The comparatively low temperatures in combination with opacity render radiative transfer inefficient. The early high luminosities driven by the heat-input via dynamic mass-accretion and rapid gravitational self-compression render young self-gravitating objects fully or largely convective.

Hayashi's classical argument to determine the structure of young stars when they first 'appear' on the sky in visual light uses that very fact. Assuming the rapid gravitational collapse to be essentially adiabatic, Hayashi demonstrated that for any assumption of the thermal structure of an initial gravitationally unstable cloud fragment the self-compression during the collapse would free enough heat to drive temperature gradients beyond convective instability resulting in fully convective, essentially isentropic interior structures. Thus, the properties of young stars of given mass would be determined by the properties of their first photospheres and the adiabats which the sub-photospheric structure would follow. For given mass and given luminosity the photosphere would determine an interior specific entropy and consequently the radius, hence an effective temperature, and for a series of luminosities a line in the Hertzsprung-Russel diagram (HRD). For given mass the only remaining question would be at *which* luminosity the objects appear. That would be determined in the *forbidden region* to the right of the Hayashi line in the HRD. That region is forbidden for a hydrostatic structure of given mass because no structure can be more than fully convective. This part of the HRD would be the home of dynamic, collapsing and accreting objects. At low luminosities and at low temperatures would be the home of the first objects that would be optically thick in the frequencies relevant for energy transfer and depart from the isothermality of the initial cloud fragments for the first time: the protostellar cores. Forbidden was this region also from a technical point of view. The collapse-flows would start convection from globally convectively stable, originally isothermal structures in a dynamical environment. But there was no suitable convection theory at hand to calculate such events.

Larson (1969) showed that the protostellar collapse would be non-adiabatic and stars would not appear high on Hayashi's line, with radii orders of magnitude larger than on the main sequence, but rather at a moderate factor above main sequence values — 2–3 for the solar case. To determine the 'height' on the Hayashi-lines, at which young stars would appear, concepts were developed to avoid dynamical complications and time-dependent convection in the forbidden region. Larson assumed that accretion-flows would deposit mass on a-piori fully convective, isentropic structures calculating their entropy value from a simplified spherical collapse. Stahler assumed that young stars would appear at the point on the Hayashi line were Deuterium-burning would set in, presumably rendering the structure fully convective. All fully convective D-burning structures would thus form a *birth-line* of stars. See Wuchterl & Tscharnuter (2003) for references and further discussion. In the following I describe a convection model that can be used in the forbidden region and I discuss applications to star-formation, young brown dwarfs, planet formation and object-classification in direct imaging searches for extrasolar planets.

2. Convection and the masses of giant planets

Giant planets most likely form in gravitationally stable, circumstellar disks - the protoplanetary nebulae. With the standard planetesimal hypothesis giant planets form via the growth and extra gravity of a condensible element core. That core becomes a large 'terrestrial planet' and gradually accumulates nebula gas in an almost static way up to the so-called *critical mass*. At the critical mass, which typically is a few to ten Earth masses, hydrodynamic events set in with an outcome depending on the interior structures of the proto giant planet's envelopes. 'Sufficiently' convective envelopes lead to the onset of rapid accretion by damping an instability found in 'radiative' protoplanetary envelopes that typically occur at lower protoplanetary nebula densities and large orbital radii, Wuchterl (1995a), see Wuchterl et al. (2000) for review. Furthermore, the masses of largely convective proto giant planets, unlike their radiative counterparts, depend on the properties of the ambient nebula, Wuchterl (1993), Ikoma et al. (2001). That makes a physically plausible, time-dependent convection theory a necessity for giant planet formation studies that aim for a quantitative understanding of planetary masses.

The radiation fluid-dynamics of giant planet formation with typical peak Mach-numbers of a few, compared to the few hundred of stellar collapse makes it an ideal test-bed for time-dependent convection. The flows of planet formation show many of the features found in stellar collapse — non-compact photospheres, strong shocks, close to hydrostatic equilibrium and free-flowing parts, but are much less extreme in terms of computational challenges as dynamic range and resolution.

3. The Kuhfuß-convection-model

Any convection model to be included in calculations of star- and planet formation must satisfy a number of conditions including time-dependence, the onset from and the decay to globally stable stratifications, robustness, and a well-defined 'stellar-evolution' limit, see Wuchterl & Feuchtinger (1998). The model of Kuhfuß (1987) was developed for applications during time-dependent phases of stellar evolution but is sufficiently general to provide a basis for stellar fluid-dynamics in general. There are two versions: (a) a one equation version with a single equation for the turbulent kinetic energy, and (b) a three equation version. In both original versions there are implementation difficulties that are sometimes solved by applying a turbulent 'seed'. But a reformulation of the *one-equation-version* for self-adaptive grids Wuchterl (1995b) allows routine inclusion into radiation fluid-dynamics without seeding thus taking full advantage of the model's ability to describe the onset of convection from and decay to flows with zero convective energy density. Furthermore, the original Kuhfuß-version fails to reproduce the solar convection zone and violates the flux-limit under highly super-adiabatic conditions, with significant radiation energy densities. The latter effect is particularly strong in supercritical protostellar accretion shocks. In a reformulation by Wuchterl & Feuchtinger (1998), the one equation model closely approximates standard mixing length theory in a static, local limit and accurately describes the solar convection zone and RR-Lyrae lightcurves, Wuchterl & Feuchtinger (1998).

The one equation model has been applied to the evolution of Pop III stars, Straka & Tscharnuter (2001), and was successfully coupled to large nuclear networks. Flaskamp (2003) showed that it also satisfies the helioseismological constraints and successfully describes the mass-dependence of overshooting for stars at the zero-age-main-sequence.

In the simplest Wuchterl & Feuchtinger (1998) version, the heart of the modified one equation model is a dynamical equation for the specific kinetic energy density, ω, of convective elements. An equation accounts for creation of eddies by buoyancy, dissipation of eddies due to viscous effects as well as eddy advection and radiative losses:

$$\frac{d}{dt}\left[\int_{V(t)} \varrho\omega\, d\tau\right] + \int_{\partial V} \varrho\omega u_{\mathrm{rel}} \cdot dA = \int_{V(t)} \left(S_\omega - \tilde{S}_\omega - D_{\mathrm{rad}}\right) d\tau \tag{3.1}$$

where the eddy kinetic energy generation-rate, the eddy dissipation-rate, the convective enthalpy flux, the reciprocal value of the mixing-length, Λ, and the time-scale for radiative eddy losses, respectively, are:

$$S_\omega = -\nabla_{\mathrm{s}} \frac{T}{P}\frac{\partial P}{\partial r}\Pi \tag{3.2}$$

$$\tilde{S}_\omega = \frac{c_{\mathrm{D}}}{\Lambda}\omega^{3/2} \tag{3.3}$$

$$j_{\mathrm{w}} = \varrho T \Pi\,, \quad \Pi = \frac{w}{T}\, u_c F_L \left[-\sqrt{3/2}\ \alpha_{\mathrm{S}}\, \Lambda \frac{T}{w}\frac{\partial s}{\partial r}\right], \tag{3.4}$$

$$\frac{1}{\Lambda} = \frac{1}{\alpha_{\mathrm{ML}}\, H_p^{\mathrm{stat}}} + \frac{1}{\beta_r r}, \quad H_p^{\mathrm{stat}} = \frac{p}{\varrho}\frac{r^2}{GM_r}, \tag{3.5}$$

$$\tau_{\mathrm{rad}} = \frac{c_p\, \kappa\, \rho^2\, \Lambda^2}{4\sigma\, T^3\, \gamma_{\mathrm{R}}^2}\,, \quad D_{\mathrm{rad}} = \frac{\omega}{\tau_{\mathrm{rad}}} \quad . \tag{3.6}$$

In the time-independent and static limit this is essentially mixing-length theory and the accuracy is assured by fitting the prescription to the Sun via a solar model. The differences to mixing length theory are an improved physical plausibility, in particular the time-behaviour and that the parametrisation is now brought into a fluid-dynamical deductive framework, (cf. Wuchterl & Feuchtinger (1998)). The Schwarzschild-Ledoux criterion is contained in the formulation via $-\partial s/\partial r = c_p/H_p(\nabla - \nabla_{\mathrm{s}})$ and $\nabla_{\mathrm{s}} = \nabla_{\mathrm{ad}}$ in the absence of energy sources and sinks inside eddies. Convectively unstable stratifications occur in this model when pressure and temperature gradients have the same sign and produce a positive value of S_ω that then contributes a source of turbulent kinetic energy, $\omega = 3/2u_c^2$ to the balance equation of turbulent kinetic energy, Eq. (3.1). u_c being the convective velocity corresponding to mixing length theory. A general problem of mixing length theory — the violation of a convective flux-limit — has been corrected as described by Wuchterl and Feuchtinger by introducing a flux-limiting function (cf. Wuchterl & Tscharnuter 2003 where the coupling terms are shown in the full set of equations for self-gravitating radiating fluids). The great advantage of that approach is that a general prescription can be used for the Sun, stellar evolution, pulsating stars, brown dwarfs, planets and protoplanets. Any calibration of parts of the convection model obtained in one astrophysical system — the mixing length parameter calibrated by the Sun, the time-dependent behaviour tested by RR-Lyrae stars — will decrease the uncertainties in applications to not-so-easy-to-observe systems such as protoplanets.

4. Cloud collapse: from stars to brown dwarfs

To determine the initial conditions for stellar evolution Wuchterl & Tscharnuter (2003) calculated the collapse of marginally gravitationally unstable, isothermal equilibrium configurations — so called Bonner-Ebert spheres — of masses ranging from stars to brown dwarfs with the fully time-dependent equations of radiation fluid-dynamics and the modified Kuhfuß-model. As in previous studies without convection, a radial temperature inversion that is characteristic for accretion processes was found. With convection included, the hydrostatic protostellar cores show a convective shell and a radiative interior from the beginning. That structure persists to the end of accretion for all masses, i.e. down into the brown dwarf regime. Accordingly all objects that start as gravitationally unstable Bonnor-Ebert-spheres have radiative interiors when all of their final mass has settled into hydrostatic equilibrium for the first time. For the case of a solar mass, that results in twice the luminosity and an effective temperature increased by 500 K compared to a classical pre-main sequence calculation starting high on the Hayashi track, Wuchterl & Tscharnuter (2003). Results are the same after one Ma, when accounting for 3D-fragmentation in a dense cluster Wuchterl & Klessen (2001). Young stars and brown dwarfs should therefore not appear on the Hayashi-lines but somewhat hotter, to their left. The off-centre temperature-maximum results in an off-centre ignition of Deuterium and persistent shell burning. For the solar case all of the Deuterium is burnt during the embedded phases of accretion and no significant amounts of Deuterium remain to cause a 'birth-line' effect during pre-main sequence evolution. For brown dwarf masses significant amounts ob Deuterium survive the accretion phase and thus they may show a slow down of their early contraction due to Deuterium contributions to their energy-budget. There are numerous further consequences for pre-main sequence evolution. High precision parameters of young stars obtained by interferometric means and high precision spectroscopy should be able to test these theoretical predictions in the near future.

5. Extreme planet formation - GQ Lupi b and HD 149 026 b

GQ Lupi is a classical T Tauri star in the Lupus star formation region with an estimated age of a few Ma. Neuhäuser et al. (2005) reported the detection of a co-moving faint and cool companion. To determine the age and the masses of the system, conventional models of stars, brown-dwarfs and planets cannot be used because they do not take into account the formation processes. At very young ages they only show the ad-hoc initial conditions that have been set to start the calculations. Thus, collapse calculations for stars and brown-dwarfs are necessary to determine the properties of the

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components of the GQ Lupi system. Neuhäuser et al. (2005) extended the Wuchterl & Tscharnuter (2003) calculations down to 13 Jupiter masses, the lower mass limit for brown dwarfs according to the working definition of the IAU working group of extrasolar planets (WGESP). Neuhäuser et al. (2005) found the companion to be less luminous and cooler than the track for the lowest mass brown dwarf. Thus, the object is of planetary mass based on the only available models accounting for the formation process and hence the only applicable theoretical model. Furthermore, Neuhäuser et al. (2005) used the modified Kuhfuß model with the Wuchterl & Tscharnuter (2003) equations and 'planet-formation' boundary conditions, Wuchterl (1991), with a particle-in-box planetesimal accretion-rate. All gravitationally stable nebulae were allowed, hence abandoning the minimum-mass concept (see Wuchterl *et al.* 2000 for a discussion of the latter). Neuhäuser et al. (2005) found planetary models — as specified above — fitting the observables of the GQ Lupi companion for 1-3 Jupiter masses. The fitting planetary models are coeval with the primary star — a consistency check — at an age of about 2 Ma and their surface gravity is consistent with the companion spectrum. Thus, the system is best modelled by an 0.7 solar mass star and a 1-3 Jupiter mass planet, the first one imaged and proved to be co-moving with its primary.

HD 149 026 is a 1.3 solar mass field star. Sato et al. (2005) discovered a transiting planetary companion with an orbital period of about 3 days, a mass of 114 earth masses and a measured radius auf 0.725 Jupiter-radii. The resulting unexpected high density can be explained by a 70 earth mass condensible element 'rocky' core. Thus providing the first key evidence for the importance of heavy elements for giant planet formation outside the solar system, as predicted by the planetesimal model. The large core can be quantitatively understood utilising a new synoptic approach to planet formation theory based on a classification and survey of all possible planetary equilibrium configurations (Broeg & Wuchterl 2006, Broeg 2006). That is somewhat analogous to constructing the main sequence by searching for all stars in the *complete equilibrium* of stellar structure theory. Applying the modified Kuhfuß-model with planetary fluid-dynamics Broeg & Wuchterl (2006) found that a planet with the properties of HD 149 026 b can form in a non-minimum-mass nebula. Furthermore, they found that the entire planet formation would be a quasi-hydrostatic process in that case.

This demonstrates the wide applicability of the modified Kuhfuß-model. One possible conclusion is that planet formation occurs in a large diversity of circumstellar disks: extreme planets can be understood with non-standard, but plausible (gravitationally stable) protoplanetary nebulae. Nebula diversity is a promising key to exoplanet diversity.

6. Convection for cosmogony - future steps

A number of applications have been shown to demonstrate the predictive power of a sufficiently simple and robust convection theory that nevertheless contains a minimum amount of physics that is necessary for fluid-dynamical studies of star and planet formation. Instead of giving a discussion I will conclude with a list of convection-related topics that would help progress in cosmogony.

(*a*) The Kuhfuß-theory, as any approach terminating a moment expansion with closure relations, needs input from more general theories. In particular, results from large eddy simulations would be much easier to implement, if they would not be expressed in quantities like effective mixing-lengths or overshooting distances but by relations for and between correlations and other statistical properties obtained from the 3D-flows.

(*b*) A low optical depth mesoscopic formalism is needed, i.e., a treatment of radiative eddy-losses outside the radiation-diffusion regime.

(*c*) An investigation of the coupling between matter and radiation in a radiation-fluid-dynamical framework under turbulent conditions is needed for convection theories based on moment expansions of the fluctuations.

(*d*) Practical spectral analysis codes with 'non-ATLAS-compatible' convection, for high quality high throughput spectral analysis.

(*e*) More helioseismology and asteroseismology for Kuhfuß-type models.

(*f*) High quality observations of young, 1–10 Ma convection dominated objects to directly determine the masses and radii of young stars.

7. Conclusion

The modified Kuhfuß-theory has been applied to problems of star and planet formation because the relevant flows put tight constraints on the convection model. These constraints were more on robustness, computational efficiency and physical plausibility than on high accuracy for a specific class of objects or physical regime. Yet the model has passed a number of standard checks and helped as a problem solver in systems as different as RR Lyrae stars and strange exoplanets. Cosmogonic studies would benefit a lot from further study of the modified Kuhfuß model in objects on the sky and comparison to detailed simulations of convection.

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Discussion

ROXBURGH: You only used the Kuhfuß model of convection - how robust are your results against changes in the convection theory ?

WUCHTERL: Collapse results are robust at least for changes in $1 < \alpha_{\rm ML} < 2$. Feuchtingers RR-Lyrae results constrain the time-dependence and indicate robustness in that regime.

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