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ARTICLE

Intergenerational transfers, differential fertility, and wealth inequality

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Abstract

Rising income and wealth inequality across the developed world has prompted a renewed focus on the mechanisms driving inequality. This paper contributes to the existing literature by studying the impact from life-cycle savings, intergenerational transfers, and fertility differences between the rich and the poor on the wealth distribution. We find that bequests increase the level of wealth inequality and that fertility differences between the rich and the poor amplify this relationship. The counterfactual exercises show that the interaction between bequests and differential fertility is quantitatively important for understanding wealth inequality in the United States.

Keywords: bequest; differential fertility; wealth inequality; savings

JEL classifications: E00; E20

1. Introduction

The literature shows that standard heterogeneous agents models struggle to replicate the magnitude of the wealth inequality observed in the data. For example, the Gini coefficient of the wealth distribution generated in a baseline Aiyagari (1994) model is only around 0.4, while the U.S. wealth Gini coefficient is close to 0.82 (see Díaz-Giménez et al. (2011)). An important part of the puzzle is that the rich save more and spend less than predicted by standard models, and consequently accumulate a large amount of wealth. According to Alvaredo et al. (2013), the top 1% of households in the U.S. hold nearly one third of the total wealth and the top 5% holds over half, an order of magnitude larger than their counterparts generated in standard models.

Why is the wealth distribution so unequal? Why do rich people hold such a high amount of wealth? An important existing explanation offered in the literature is from De Nardi (2004), who emphasizes the role of bequests and intergenerational links. She finds that the rich are much more likely to leave bequests to their children compared to their poorer counterparts, even after accounting for the relative wealth between the two groups. Based on this finding, she created a model incorporating bequests into the utility function as a luxury good, and finds that this model is capable of accounting for the high concentration of wealth in the data, and that bequeathing behaviors are important in shaping the distribution of wealth. However, the De Nardi (2004) model assumes an identical fertility rate among the population, and thus abstracts from the fact that the poor tend to have more children than the rich, a dimension of heterogeneity we argue is relevant for understanding the wealth distribution. In this paper, we contribute to the literature by extending the De Nardi (2004) model to incorporate differential fertility choice among the population, and analyze the implication of differential fertility for the wealth distribution through its interaction with the bequest mechanism.

Economists have long argued that there exists an inverse relationship between income and fertility. For instance, Jones and Tertilt (2008) document a strong negative relationship between income and fertility choice for all cohorts of women born between 1826 and 1960 in the U.S. census data. They estimate an overall income elasticity of fertility of about −0.38. A recent literature in quantitative macroeconomics has argued that this negative relationship between income and fertility has important implications for some key macroeconomic phenomena such income inequality and growth.[1] In this paper, we explore its implication for the wealth distribution, and argue that this significant fertility difference between the poor and the rich can amplify the impact of bequests on wealth inequality, because not only do rich parents leave a greater amount of bequests than their poorer counterparts, but the children of rich parents have fewer siblings to share their bequests with relative to the children of poor parents.

To capture the interaction between differential fertility and bequests, and to assess its quantitative importance for understanding the wealth distribution, we develop a general equilibrium overlapping-generations (OLG) model with the "warm-glow" bequest motive (similar to that used in De Nardi (2004)) and differential fertility. Using a version of our model calibrated to the U.S. economy, we find that assuming away the fertility difference between the rich and the poor reduces the Gini coefficient of wealth from 0.78 to 0.74 in the model, with the result largely driven by the changes in the top wealth shares (i.e., 10%, 5%, 1%). We also find that shutting down the fertility difference together with the bequest motive can reduce the wealth Gini further to 0.69. In addition, we find in our model that anticipated bequests crowd out life-cycle savings, which implies that intergenerational transfers can lead to less capital formation. In sum, this paper finds that pairing bequest motive with differential fertility is quantitatively important for explaining the saving behaviors of the rich and the consequent high level of wealth inequality.

1.1 Literature review

Ever since heterogeneous agent macroeconomic models have been introduced to the macroeconomics literature, numerous papers have used this class of models to explain the causes and mechanisms behind wealth inequality.[2] As surveyed by De Nardi (2015), there have been many variations of the heterogeneous agent model which introduce various mechanisms to better match the magnitude of wealth inequality observed in the data, such as preference heterogeneity, entrepreneurship, high earnings risk for the top earners, transmission of bequests across generations, and others.[3] Among these, our paper relates to the literature espousing bequest transmission across generations as a main mechanism behind wealth inequality.

The two papers in this literature closest to ours in spirit are De Nardi (2004) and Knowles (1999). De Nardi (2004) uses a quantitative, general equilibrium, OLG model in which bequests and ability link parents and children. The element in which our papers differ is in our treatment of fertility. In De Nardi (2004), each agent has the same number of children. In our model agents have a different number of children depending on the income, impacting the results in interesting ways. Our model is also close in spirit to Knowles (1999), who uses a two period model to show the importance of fertility to inequality. In his model, there is no retirement period, which means savings that occur in his model are solely for the purpose of bequests. In contrast, the agents in our model must save for their own retirement on top of bequests. Therefore, our model captures the dynamic interaction between life-cycle savings and anticipated bequests. We show this interaction is quantitatively important for understanding the wealth distribution. In addition, our model differs from Knowles (1999) in terms of the choice of the bequest motive. While bequests are assumed to be motivated by altruism in the Knowles (1999) model, we adopt the "warm-glow" bequest motive based on the empirical literature we will discuss below.

It is well-known in the literature that intergenerational transfers account for a large fraction of wealth accumulation.[4] However, the literature has been at odds as to how to model bequest motives, specifically whether bequests are motivated by altruism. Altonji et al. (1992) found that the division of consumption and income within a family are codependent, indicating that perfect altruism does not apply to operative transfers. Other studies show that an increase in parental resources coupled with a decrease in child consumption does not lead to a corresponding increase in transfers (Altonji et al. (1997) and Cox (1987)). Altonji et al. (1997) find a one dollar transfer from child to parent results in only a 13 cent donation from parent to child, which should be the full dollar under perfect altruism. Wilhelm, 1996) finds that siblings generally receive equally divided inheritances, rather than the size of the inheritance being dependent on relative income as perfect altruism would predict.[5] Based on these empirical findings, multiple recent papers have assumed an alternative bequest motive: the warm-glow motive.[6] That is, parents derive utility from giving while not caring directly about the well-being of the recipient. In addition, motivated by the highly skewed distribution of bequests, these papers incorporate leaving bequests into the utility function as a luxury good, allowing for rich parents to value bequests relatively more. Following the tradition in these papers, we also adopt the "warm-glow" motive and assume bequests are a luxury good.

Our paper also relates to a growing number of papers that have shown that allowing for transfer of ability and human capital across generations is also an important element for understanding inequality. These studies include Kotlikoff and Summers (1981), Knowles (1999), De Nardi (2004), De la Croix and Doepke (2004), De Nardi and Yang (2016), Daruich and Kozlowski (2020) and others. Of special note, Lee, Roys, and Seshadri (Lee et al. 2024) find that parental education is positively related to their children's earnings, thereby creating a virtuous cycle for the wealthiest and a vicious cycle for the poorest.

The rest of paper is organized as follows. In Section 2, we describe the model and its stationary equilibrium. In Section 3, we calibrate a benchmark model. In Section 4, we discuss the main quantitative results and conclude in Section 5.

2. The model environment

Consider an economy inhabited by OLG of agents who can live up to 14 periods, with one period corresponding to 5 years. Agents are endowed with 1 unit of time per period that can be divided between work and time spent raising a child. Population grows at an annual rate of n%. There is a government that runs a pay-as-you-go social security system, the details of which are explained in the following section.

2.1 Consumer's problem

Agents begin their economic life at the age of 20, which corresponds to period $t = 1$ in the model.[7] In the first 3 periods ($t = 1, 2, 3$), agents are young adults in the labor market. At age 35 ($t = 4$), agents' children are born in which the number of children $N(\psi)$ is exogenously determined by agent's permanent ability ψ such that we can account for the differential fertility across income groups.[8] By age 55 ($t = 8$), agents' children have left home and agents' parents have died. Working agents pay a payroll tax τ_s on their wage. These tax proceeds are used to finance social security payment $SS(\psi)$ to the retirees, which is an increasing function of their permanent ability. Retirement takes place at age 65 ($t = 10$) and each agent faces positive probability p_t of surviving until they hit the maximum possible age of 90, after which they die for sure. This implies that from age 35 ($t = 4$) until age 55 ($t = 8$), agents face the possibility of receiving bequest from their parents, conditional on their parents having positive amount of wealth at the time of death. In the event that an agent is to receive bequest, the bequest amount will be evenly distributed among all the

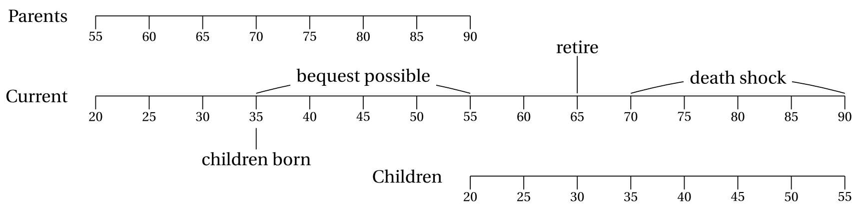

Figure 1. Sequence of events for current generation.

agent's siblings. Figure 1 shows how different generations overlap with each other during the economically meaningful ages of 20 to 90.

Period utility function takes the CRRA form, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and β is the discount factor for one period (equivalent to five years in real time). Labor productivity of agent at age t is given by $\psi\theta_t\lambda_t$, in which ψ is the inherited permanent ability, θ_t is the deterministic life-cycle earnings profile, and λ_t is the stochastic shock. The shock λ_t evolves according to the following simple AR(1) process:

$$\log(\lambda_t) = \upsilon \log(\lambda_{t-1}) + \epsilon_\lambda$$

where

$$\epsilon_\lambda \sim N\left(0, \sigma_\lambda^2\right), \quad \text{i.i.d.}$$

We assume that children inherit the permanent ability level ψ from their parents according to an AR(1) process:

$$\log(\psi) = \rho \log\left(\psi_p\right) + \epsilon_\psi$$

where

$$\epsilon_\psi \sim N\left(0, \sigma_\psi^2\right), \quad \text{i.i.d.}$$

in which ψ_p is the parental ability and ρ is the intergenerational persistence of productivity. We discretize the AR(1) process into an 11-state Markov chain using the Tauchen and Hussey (1991) algorithm, and the corresponding transition matrix we obtain is denoted by $M\left[\psi, \psi'\right]$.

Hence, the agent's state variables are given by age (t), asset or wealth (a), ability (ψ), the productivity shock (λ), and their parent's state variables S_p.[9] The reason why number of children is not one of the state variables for neither the current generation nor the parent generation is because the number of children would exogenously equal $N(\psi)$ and the number of siblings (which is equivalent to the number of children the parent generation had) would exogenously be set to $N\left(\psi_p\right)$.

2.1.1 Age 20 to 30

These young workers do not have children yet and do not expect to receive any bequest from their parents. Hence, their value function is

$$V\left(t, a, \psi, \lambda, S_p\right) = \max_{c,a'} \left\{u(c) + \beta \mathbb{E}\left[V\left(t+1, a', \psi, \lambda', S_p'\right)\right]\right\}$$

subject to

$$\begin{aligned} c + a' &\leq \psi\theta_t\lambda w(1-\tau_s) + (1+r)\,a \\ a' &\geq 0 \\ S_p' &= g\left(S_p\right) \end{aligned}$$

We do not allow for any inter-vivos transfers and impose a strict non-negative borrowing constraint, which implies that the agents start the model at age 20 ($t=1$) with initial wealth of 0. w is the equilibrium wage per unit of effective labor. Expectation for the continuation value of the Bellman equation is taken with respect to the productivity shocks λ' and λ'_p, conditional on λ and λ_p. The last equation is the law of motion for parent's state variables, where $g(\cdot)$ is the updating rule for parent's state variables.

2.1.2 Age 35 to 50

Starting in age 35 ($t=4$), agents not only have children according to the exogenous fertility function $N(\psi)$, but they also face the possibility of receiving a bequest in the event that their parents die. Let p_{t+7} denote the probability that the parent of an agent in period t would survive to next period, since the parent's age t_p will always equal $t_p = t+7$. In addition, let $V^b(t, a, \psi, \lambda)$ be the value function of agents who received bequest and their parents are no longer around. Hence, the value function for agents in this age range whose parents were still alive at the beginning of the period would be

$$V\left(t, a, \psi, \lambda, S_p\right) = \max_{c,a'} \left\{ u(c) + \beta p_{t+7} \mathbb{E}\left[V\left(t+1, a', \psi, \lambda', S'_p\right)\right] + \beta\left(1-p_{t+7}\right)\mathbb{E}\left[V^b\left(t+1, a' + \frac{a'_p}{N\left(\psi_p\right)}, \psi, \lambda'\right)\right]\right\}$$

subject to

$$\begin{aligned} c + a' &\leq \psi\theta_t\lambda w(1-\tau_s)\left(1-\gamma_c N(\psi)\right) + (1+r)\,a \\ a' &\geq 0 \\ S'_p &= g\left(S_p\right) \end{aligned}$$

in which the continuation value of the Bellman equation accounts for the fact that there is $\left(1-p_{t+7}\right)$ probability that the parents die, in which case the agent receives whatever asset that the parent still had at the time of death divided by the number of siblings as the bequest.

In the budget constraint, γ_c is the total cost per child per parent. Note that $\gamma_c = \gamma_t + \gamma_g$. That is, the total childcare cost includes time cost, γ_t and goods cost, γ_g. Thus, $(1-\gamma_t N(\psi))$ simply represents the total amount of time available to be allocated to labor. This implies that $\psi\theta_t\lambda\,(1-\gamma_t N(\psi))$ is the total amount of effective labor supplied, with w measuring the equilibrium real wage per effective unit of labor. Note that because some child costs are in terms of time, the opportunity cost of raising a child is higher for parents with higher ability ψ that tend to have higher labor earnings, as is expected and reflected in the data.

The value function for agents who already received bequest from their parents because they passed away would be

$$V^b(t, a, \psi, \lambda) = \max_{c,a'} \left\{ u(c) + \beta\mathbb{E}\left[V^b\left(t+1, a', \psi, \lambda'\right)\right]\right\}$$

subject to

$$\begin{aligned} c + a' &\leq \psi\theta_t\lambda w(1-\tau_s)\left(1-\gamma_c N(\psi)\right) + (1+r)\,a \\ a' &\geq 0 \end{aligned}$$

2.1.3 Age 55 and 60

Age 55 ($t = 8$) and 60 ($t = 9$) are when parents have passed away and children have left the nest, yet agents are still in labor force. Hence, their value function is the following:

$$V^b(t, a, \psi, \lambda) = \max_{c,a'} \left\{ u(c) + \beta \mathbb{E} \left[V^b \left(t+1, a', \psi, \lambda' \right) \right] \right\}$$

subject to

$$c + a' \leq \psi \theta_t \lambda w (1 - \tau_s) + (1 + r)\, a$$
$$a' \geq 0$$

2.1.4 Age 65 and above

Agents are now retired, so they no longer work and simply consume out of their savings and social security payments, $SS(\psi)$, which is an increasing function of the agent's permanent ability. The value function for agents in this age range would be

$$V^b(t, a) = \max_{c,a'} \left\{ u(c) + \beta p_t V^b \left(t+1, a' \right) + \beta \left(1 - p_t \right) \phi_1 \left(1 + \frac{a'}{\phi_2} \right)^{1-\sigma} \right\}$$

subject to

$$c + a' \leq (1 + r)\, a + SS(\psi)$$
$$a' \geq 0$$

where p_t is the probability of surviving to next period, which decreases with age and $p_{14} = 0$ so that at age 85 ($t = 14$), the agent knows for sure that death awaits at age 90.

With $\left(1 - p_t\right)$ probability, the agent does not make it alive to the next period, and leaves bequest to her children, from which the parents do receive utility. Here we follow De Nardi (2004) and assume that parents have "warm glow" motive, where they enjoy giving to their children but do not directly care about the children's well-being; in addition, bequest is assumed to be a luxury good. As we reviewed in the introduction, this assumption is consistent with a sizable empirical evidence. The term ϕ_1 measures the relative weight placed on the bequest motive, while ϕ_2 measures the extent to which bequests are a luxury good.

2.2 Firm's problem

Firms are identical and act competitively. Their production technology is Cobb–Douglas, which combines aggregate capital K and aggregate labor L to produce output Y as follows

$$Y = zK^{\alpha} L^{1-\alpha},$$

where α is the capital share and z is the total factor productivity (TFP).

The profit-maximizing behaviors of firms imply that

$$r = z\alpha K^{\alpha-1} L^{1-\alpha} - \delta$$

and

$$w = z(1 - \alpha) K^{\alpha} L^{-\alpha},$$

where δ represents the capital depreciation rate.

2.3 Stationary equilibrium

We look for a stationary equilibrium in which the factor prices w and r, and the age-wealth distribution are constant over time. Let $\Phi\left(t, a, \psi, \lambda, S_p\right)$ represent the population distributions of agents across the state variables. A stationary equilibrium (steady state) of this economy consists of the following:

1. Given prices w and r, the value functions $V\left(t, a, \psi, \lambda, S_p\right)$ and $V^b\left(t, a, \psi, \lambda\right)$ are solutions to the utility maximization problem, and $c\left(\cdot\right)$ and $a'\left(\cdot\right)$ are the associated optimal policy rules with respect to consumption this period and asset next period.
2. The prices w and r are equal to its marginal product such that

$$r = z\alpha K^{\alpha-1}L^{1-\alpha} - \delta$$
$$w = z(1-\alpha)K^{\alpha}L^{-\alpha}$$

3. All markets clear such that

$$K' = \sum_{t=1}^{14}\int_a\int_\psi\int_\lambda\int_{S_p} a'\left(t, a, \psi, \lambda, S_p\right) d\Phi\left(t, a, \psi, \lambda, S_p\right)$$
$$L' = \sum_{t=1}^{3}\int_a\int_\psi\int_\lambda\int_{S_p} \psi\theta_t\lambda d\Phi\left(t, a, \psi, \lambda, S_p\right)$$
$$+ \sum_{t=4}^{7}\int_a\int_\psi\int_\lambda\int_{S_p} \psi\theta_t\lambda\left(1-\gamma_t N\left(\psi\right)\right) d\Phi\left(t, a, \psi, \lambda, S_p\right)$$
$$+ \sum_{t=8}^{9}\int_a\int_\psi\int_\lambda\int_{S_p} \psi\theta_t\lambda d\Phi\left(t, a, \psi, \lambda, S_p\right)$$

4. Social security system is self-financing. That is, the following budget constraint holds:

$$SS(\psi)\sum_{t=10}^{14}\int_a\int_\psi d\Phi(t, a, \psi) = \tau_s\sum_{t=1}^{3}\int_a\int_\psi\int_\lambda\int_{S_p} \psi\theta_t\lambda w d\Phi(t, a, \psi, \lambda, S_p)$$
$$+ \tau_s\sum_{t=4}^{7}\int_a\int_\psi\int_\lambda\int_{S_p} \psi\theta_t\lambda w(1-\gamma_t N(\psi))d\Phi(t, a, \psi, \lambda, S_p)$$
$$+ \tau_s\sum_{t=8}^{9}\int_a\int_\psi\int_\lambda\int_{S_p} \psi\theta_t\lambda w d\Phi(t, a, \psi, \lambda, S_p)$$

5. The distribution Φ is stationary over the state variables.

The rest of the paper focuses on stationary equilibrium analysis. Since analytical results are not obtainable, numerical methods are used to solve the model.

3. Calibration

We calibrate the model to match some of the key moments of the U.S. economy. The main parameter values and their sources are summarized in Table 1. As described in detail below, our calibration strategy consists of two stages: some of the standard parameter values are predetermined based on previous studies or independent estimates, while the rest are simultaneously chosen to match some recent key empirical moments in the U.S. economy. In the rest of the section, we first describe the details of our calibration, and then evaluate the model fit by reviewing the main properties of the calibrated model.

Table 1. The benchmark calibration

Parameter	Value	Source
z	1.0	Normalization
σ	2.0	Macro Literature
α	0.36	Macro Literature
γ_t	0.15	Haveman and Wolfe (1995)
γ_g	0.20	Sommer (2016)
ρ	0.677	Zimmerman, 1992)
υ	0.985	Storesletten et al. (2004)
σ_λ^2	0.022	Storesletten et al. (2004)
θ_t	see text	Hansen (1993)
p_t	see text	Bell et al. (1992)
$N(\psi)$	Table 2	Jones and Tertilt (2008)
n	0.012	De Nardi (2004)
τ_s	12.4%	Hosseini et al. (2021)
Parameter	Value	Targeted Moment(s)
δ	0.06	capital-output ratio: 3.0
β	0.9489	annual interest rate: 6%
ϕ_1	−18.1	bequests to GDP ratio : 2.4–4.7% of output
ϕ_2	12.1	avg. bequest at the bottom 30% to the avg. bequests: 0.11
σ_ψ^2	0 .36	earnings distribution

3.1 Demographics

One period in our model is equivalent to 5 years in real time. Individuals begin making economic decisions when they are 20 years old, which is equivalent to $t = 1$ in model age. They retire at 65 years old and die at age 90 for sure. The conditional survival probabilities p_t for retired agents are calibrated to the survival probabilities estimated from Bell et al. (1992). Annual population growth rate is assumed to be 1.2%, consistent with the U.S. data (De Nardi (2004)).

3.2 Income process, permanent ability and fertility

We use the estimates from Hansen (1993) for the calibration of θ_t, the deterministic age profile of labor productivity. As for the stochastic income shock λ, we approximate the distribution by a 3-state Markov chain using the Rouwenhorst, 1995) algorithm, with the values of persistence υ set to 0.985 and variance σ_λ^2 set to 0.022 following the estimates in Storesletten et al. (2004).[10]

Since the value of the intergenerational persistence parameter of the the AR(1) process for parental ability, ρ, is less than 0.9, we use the the Tauchen and Hussey (1991) method to discretize it into 11 states .[11] We set its value to 0.677 based on Zimmerman, 1992). The variance of shock to permanent ability, σ_ψ^2, is chosen to match the earnings distribution. This leads to σ_ψ^2 value of 0.36. In addition, to match the wealth shares at the very top of the distribution, we scale up the top three grid points of ability distribution. The resulting values for each state of permanent ability and the corresponding transition matrix are reported in Table 2.[12]

In the benchmark calibration, we follow the approach in Jones and Tertilt (2008) and use the "Children Ever Born" to a woman as the fertility measure. To properly capture the negative income-fertility relationship, we assume that $N\left(\psi_p\right)$ takes the form, $N\left(\psi_p\right) = \gamma_1 \psi_p^{\gamma_2}$, with $\gamma_1 > 0$

Table 2. Permanent ability and its transition matrix

Ability Group i	1	2	3	4	5	6	7	8	9	10	11
ψ_i	0.04	0.09	0.18	0.32	0.57	1.0	1.75	3.08	7.53	14.32	30.35
Trans. Prob(ψ,ψ')	1	2	3	4	5	6	7	8	9	10	11
1	0.14	0.41	0.33	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
2	0.02	0.20	0.40	0.28	0.08	0.01	0.00	0.00	0.00	0.00	0.00
3	0.00	0.06	0.27	0.38	0.22	0.06	0.01	0.00	0.00	0.00	0.00
4	0.00	0.01	0.11	0.32	0.35	0.16	0.03	0.00	0.00	0.00	0.00
5	0.00	0.00	0.03	0.18	0.36	0.30	0.11	0.02	0.00	0.00	0.00
6	0.00	0.00	0.01	0.07	0.24	0.37	0.24	0.07	0.01	0.00	0.00
7	0.00	0.00	0.00	0.02	0.11	0.30	0.36	0.18	0.03	0.00	0.00
8	0.00	0.00	0.00	0.00	0.03	0.16	0.35	0.32	0.11	0.01	0.00
9	0.00	0.00	0.00	0.00	0.01	0.06	0.22	0.38	0.27	0.06	0.00
10	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.28	0.40	0.20	0.02
11	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.10	0.33	0.41	0.14

Data source: Krueger et al. (2016)

and $\gamma_2 < 0$. Note that the value of γ_2 is exactly equivalent to the income elasticity of fertility, and thus we set it to -0.20 based on the estimate of Jones and Tertilt (2008). The value of γ_1 is then chosen so that the model implied average fertility is equal to the replacement rate, that is, 1 child per parent. The time cost of children, γ_t, is assumed to be 15% of parental time per child based on the empirical estimates of Haveman and Wolfe (1995). The goods cost of children, γ_g, is taken from Sommer (2016), which estimates it to be 20% of the earnings per parent. Therefore, the overall cost of children, γ_c, in our model is 35% of the earnings of each parent.

3.3 Preference and technology

The subjective discount factor β is calibrated to a value of 0.9489 to generate an annual interest rate of 6%, as in De Nardi (2004). The risk aversion parameter, σ, is set to 2, a standard value used in the existing macro literature. The bequest motive parameters ϕ_1 and ϕ_2 are jointly calibrated to match two moments: (1) the bequest to GDP ratio, and (2) the ratio of average bequest at the bottom 30% of the distribution to average bequest of the population. The value of ϕ_1 controls the intensity of bequest motive, with a higher value showing higher intensity. A positive value of ϕ_2 implies that bequests are luxury goods. Wang (2016) estimates that between 1996 and 2012, the bequest to GDP ratio in the United States ranged between 2.4% and 4.7%. We calibrate the value of ϕ_1 to generate a bequest to GDP ratio of 4.04%, within the range of empirical estimates provided in Wang (2016).[13] The resulting value for ϕ_1 is -18.1. According to Hurd and Smith (1999), the ratio of average bequest at the bottom 30% of the distribution to the average bequests is approximately 0.11, which is used as the target moment to calibrate ϕ_2. With ϕ_2 value of 12.1 the model generated counterpart to this moment is 0.12.

The capital share α is set to 0.36, a value widely used in the macro literature. The annual capital depreciation rate δ is set to 6% to generate capital-output ratio of 2.98, which is close to its data counterpart, 3. The value of TFP parameter, z, is normalized to one.

3.4 Government

The U.S. social security program is financed by a payroll tax rate of 12.4%, and thus we set $\tau_s =$ 0.124. Social security payment $SS(\psi)$ is a non-linear function of individuals' permanent ability,

Table 3. Benchmark model statistics vs data moments

Moment	Model	Data
Capital-output ratio	3.0	3.0
Aggregate bequest to output ratio	4.04%	2.4–4.7%
Average bequests of the bottom 30 percent to average bequests	0.12	0.11
Gini coefficient of the US earnings distribution	0.46	0.43
Average number of children per household	2	2
Income elasticity of fertility	−0.20	−0.20 to −0.21

Table 4. Earnings distribution: benchmark economy vs data

Percentile	Bottom 60%	60−80%	Top 20%	Top 10%	Top 5%	Top 1%	Gini Coef.
Data	0.29	0.23	0.49	0.32	0.21	0.08	0.43
Benchmark Model	0.27	0.22	0.52	0.35	0.25	0.08	0.46

Data source: Díaz-Giménez et al. (2011)

Table 5. Wealth distribution: benchmark economy vs data

Percentile	Bottom 60%	60–80%	Top 20%	Top 10%	Top 5%	Top 1%	Gini Coef.
Data	0.05	0.11	0.83	0.71	0.60	0.34	0.82
Benchmark Model	0.07	0.13	0.80	0.66	0.52	0.28	0.78

Data source: Díaz-Giménez et al. (2011)

and it is set to mimic the actual rules of the U.S. social security program. Specifically, we use the social security benefit formula

$$\widetilde{SS}(\psi) = \begin{cases} 0.9\bar{e}(\psi), & \bar{e}(\psi) \leq 0.2\bar{e}_a, \\ 0.18\bar{e}_a + 0.33\left(\bar{e}(\psi) - 0.2\bar{e}_a\right), & 0.2\bar{e}_a < \bar{e}(\psi) \leq 1.25\bar{e}_a, \\ 0.5265\bar{e}_a + 0.12\left(\bar{e}(\psi) - 1.25\bar{e}_a\right), & 1.25\bar{e}_a < \bar{e}(\psi), \end{cases}$$

where $\bar{e}_a$ is the average earnings in the economy, and $\bar{e}(\psi)$ is the average earnings of individuals with permanent ability ψ.[14] We rescale the social security payments so that the social security program is self-financing at the steady state.

3.5 Properties of the benchmark model

Table 3 contains some key statistics of the benchmark economy together with their data counterparts. As can be seen, our calibrated benchmark model matches the key empirical moments from the US economy fairly well. The model also matches the earnings distribution well (Table 4), although it generates slightly higher gini coefficient of earnings.

The wealth distribution generated by our benchmark model is reported in Table 5. Overall, our model does a fairly accurate job of matching the actual distribution of wealth in the U.S, especially towards the very top. In the data, those at the bottom 60% of the wealth distribution only hold 5% of total wealth. The model generated share of the bottom 60% is 7%. The richest 20% in the US hold 83% of wealth. The model counterpart of this share is 80%. In addition, the calibrated benchmark economy generates the wealth shares held by the top 1%, 5%, and 10% that are close to the data. The Gini coefficient of wealth generated in the model is 0.78, reasonably close to its data counterpart.[15]

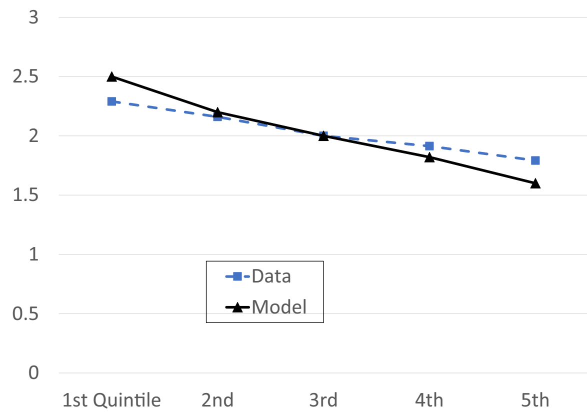

Figure 2. Fertility rate by wealth quintile: model vs data.
Data source: 2014 HRS data. The fertility rates from the data sample are scaled so that the median value matches the model.

Figure 2 displays the number of children per household across the wealth distribution in the model together with the data counterparts. In the benchmark model, the average number of children is 2.5 among the households in the 1st wealth quintile. This number declines as wealth increases, and the households in the 5th wealth quintile have 1.6 number of children on average. As shown in the same figure, the fertility rate-wealth relationship in the model matches the data counterparts reasonably well.[16]

4. Decomposing factors contributing to wealth inequality

To understand the role of differential fertility and bequests in shaping the U.S. wealth inequality, we conduct three counter-factual computational exercises. In the first counter-factual exercise, we impose identical fertility across all income (ability) groups to show the effects of differential fertility on the distribution of wealth. In the second counter-factual exercise, we shut down the bequest motive to highlight the impact of bequests on wealth inequality. In the third experiment, we shut both the channels to see their combined impact on wealth inequality.

From these counter-factual exercises, two things become clear. We find that bequests contribute to the level of wealth inequality, and fertility differences between the rich and the poor amplify this effect, especially for the far right tail of the wealth distribution. Therefore, the interaction between differential fertility and bequests is quantitatively important for fully understanding wealth inequality in the US.

4.1 Counter-factual exercise I: identical fertility

To highlight the important role of how the fertility differences across the income groups amplify the impact of bequests on wealth inequality, we consider a counter-factual exercise in which fertility is assumed to be identical across the income distribution. That is, we force everyone in the model to have the same fertility choice, 1 child per parent, and re-run the model using exactly the same parameter values as in the benchmark model.

The main results from this counter-factual exercise are reported in Table 6 (in the third row). We see that overall wealth inequality, measured by Gini coefficient, declines from 0.78 in the benchmark model to 0.74 in the identical fertility experiment (approximately an 5% decline), signaling that differential fertility increases wealth inequality. Recall that the interaction between fertility and bequest motive is a key mechanism in our model. Since more bequests are left by the

Table 6. Wealth distribution: benchmark vs counter-factual exercises

Percentile	Bottom 60%	60–80	Top 20	Top 10	Top 5	Top 1	Gini Coef.
Data	0.05	0.11	0.83	0.71	0.60	0.34	0.82
Benchmark Model	0.07	0.13	0.80	0.66	0.52	0.28	0.78
Identical Fertility	0.09	0.15	0.76	0.60	0.45	0.24	0.74
No Bequests	0.11	0.18	0.72	0.54	0.39	0.16	0.70
Iden. Fert. & No Beq.	0.11	0.19	0.70	0.52	0.37	0.15	0.69
No Beq. & Acc. Beq. to All	0.11	0.18	0.72	0.54	0.39	0.16	0.70
No Beq. & Acc. Beq. to Newborns	0.16	0.18	0.66	0.49	0.35	0.15	0.63
Same Children Cost	0.08	0.14	0.78	0.63	0.49	0.26	0.76

very rich in the US, one would expect that the impact of differential fertility on wealth inequality could be concentrated toward the top end of the distribution. This is precisely what we find through this experiment. Observe in Table 6 that, with identical fertility, the wealth share of top 1% declines from 28% to 24% and that of the top 10% declines from 66% to 60%, while the wealth share of the bottom 60% increases from 7% to 9%.

The fundamental reason why the very rich hold less wealth at the steady state in this counter-factual exercise is that the artificial increase in fertility in the counter-factual experiment forces the very rich to share their parent's wealth with significantly greater number of siblings. An important mechanism that contributes to wealth inequality in our benchmark model is that rich parents leave much bigger bequest compared to their poorer counterparts, but on top of that, the children of those rich parents also have less siblings to share that bequest with. In other words, children from rich households not only have a bigger pie to share, they have less siblings to share it with. Imposing counter-factual identical fertility to our model shuts down that channel, and we observe that has a quantitatively important impact, especially on the top 1%.

4.2 Counter-factual exercise II: no bequests

To highlight the role of bequests on wealth inequality, we now consider another counter-factual exercise in which we remove the bequest motive completely. That is, everything else is same as in the benchmark model except that the bequest motive is absent in this experiment. This allows us to parse out the effects of fertility and bequests. Specifically, we set ϕ_1 equal to zero. Results from this counter-factual exercise are shown in the fifth row of Table 6. As can be seen, the bequest motive plays an important role in generating wealth inequality. When bequest motive is removed, the Gini coefficient declines further, dropping to 0.70 from 0.78 in the benchmark model. Consistent with the existing findings in the literature, the impact of bequests is larger towards the very top of the wealth distribution. The wealth share of the top 1% declines from 28% to 16% and that of the top 10% declines from 66% to 54%, while the wealth share of the bottom 60% increases from 7% to 11%. The reason why we see larger impact towards the top is that bequests are luxury goods in our model and thus the rich leave proportionally more bequests than the poor.

4.3 Counter-factual exercise III: identical fertility and no bequests

We run another experiment in which we shut the differential fertility channel and bequest channel simultaneously. This allows us to examine the combined impact of the two channels in shaping wealth inequality. The sixth row of Table 6 presents the results, which show that when both the channels are shut, wealth inequality declines even further driven by a significant decline in the wealth share of the very rich. The Gini coefficient of wealth declines from 0.78 in the benchmark model to 0.69. The wealth share of the top 1% declines from 28% to 15% and that of the top 5%

declines from 52% to 37%, while the wealth share of the bottom 60% increases from 7% to 11%. This experiment shows that the interaction between fertility and bequests is an important factor in explaining wealth concentration towards the top of the distribution in the US.

4.4 The role of children cost and accidental bequests

In our model, differential fertility affects wealth inequlity in two ways. First, it affects how much bequests children receive. Second, the number of children affects total children cost parents incur. We showed in the first counter-factual experiment that the former is an important factor. To assess the role of the latter, we run a counter-factual experiment in which we assign all parents the same childcare cost irrespective of the number of children they have. That is, everything else is same as in the benchmark model except that each parent now incurs the average childcare cost, independent of the number of children they have. This allows the children to continue splitting the bequests as in the benchmark model, but both rich and poor parents incur the same total children cost. As the last two of Table 6 shows, doing so reduces Gini coefficient from 0.78 in the benchmark model to 0.76 and the wealth share of top 1% from 28% to 26%. The wealth share of top 5% drops from 52% in the benchmark model to 49% in this experiment. This experiment shows that children cost is a reason why differential fertility explains wealth inequality in the US.

Since premature death is allowed in the model, some of the bequests are accidental. Even if the bequest motive is removed as in the counter-factual exercise II, accidental bequests could still affect wealth inequality. To assess the impact of this channel, we run a counter-factual experiment in which we remove bequest motive and equally distribute the accidental bequests to all alive population as a lump-sum payment. Doing so prevents the children of rich parents from inheriting larger accidental bequests than the children of the poorer parents. In the sixth row of Table 6, we see that the results are not much different from the results of the fourth row ("No Bequests" row), in which only bequest motive is removed from the benchmark model. Thus, this experiment shows that accidental bequests do not significantly change our results. On the other hand, if accidental bequests are distributed as lump-sum payment to newborns the wealth inequality declines further.

5. Conclusion

This paper contributes to the literature on the causes of wealth inequality by studying the impact from savings, intergenerational transfers, and fertility differences between the rich and the poor on the wealth distribution. Our results show that bequests increase the level of wealth inequality and that fertility differences between the rich and the poor amplify this relationship. Overall, we find that incorporating differential fertility significantly improves the model's capability of matching wealth inequality in the US economy. In addition, we find that properly modeling the bequest motive and the bequest distribution is important for capturing the amplifying role of differential fertility.

We conclude the paper by drawing attention to a few potentially important issues from which this paper has abstracted. For instance, we have abstracted from means-tested social insurance programs as well as medical expenses, which can be relevant for saving and wealth accumulation decisions, especially for the relatively poor. In addition, we do not model the human capital investment in children, and thus do not capture the well-known quality-quantity tradeoff of children facing parents. We leave them for future research.

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Disclaimer. The views represented are those of the author and not necessarily those of the United States Department of the Treasury or the United States Government.

Notes

1 See Caucutt et al. (2002), Greenwood et al. (2003), De La Croix and Doepke (2003), De la Croix and Doepke (2004), Zhao, 2011), Zhao (2014), among others.
2 The heterogeneous agent macroeconomics literature can be dated back to Bewley, 1986), Imrohoroglu (1989), Huggett (1993), Aiyagari (1994), and others.
3 The literature on the causes of wealth inequality includes, among others, Krusell and Smith (1998), Heer (2001), Suen (2014), Cagetti and De Nardi (2006), Castaneda et al. (2003), Knowles (1999), De Nardi (2004), De Nardi and Yang (2016), and Imrohoroglu, among others.
4 For instance, see Kotlikoff and Summers (1981), Gale and Scholz (1994), among others.
5 Note that the nature of intergenerational links can be different in developing countries that feature different institutions and less generous public insurance. For instance, Imrohoroglu and Zhao (2018) find in the Chinese data that intergenerational transfers are highly dependent on the financial and health states of parents, suggesting strong altruism between parents and children.
6 See De Nardi (2004), De Nardi and Yang (2016), among others.
7 Prior to age 20, agents are are assumed to be children who do not make any economic decisions on their own. During this period, agents simply impose time and goods costs to their parents.
8 The assumption of giving birth at age 35 is partly technical. This assumption allows us to cover most of the life cycle and meanwhile avoids the complication of multi-generation families. The same assumption has been made in several existing studies for the same reason, such as Fuster et al. (2003), Fuster et al. (2007), Imrohoroglu and Zhao (2018), Imrohoroglu and Zhao (2020). It is also important to note that the timing of fertility differ across income groups. Caucutt et al. (2002) capture this dimension of heterogeneity as well as differential fertility in an equilibrium search model with marriage and fertility, and study the interactions between wage inequality and differential marriage and fertility decisions of young women.
9 S_p would contain age t_p, asset holding a_p, ability ψ_p, and the stochastic productivity shock λ_p. It is important to note that the state variables of the grandparents S_{pp} would not be part of the parent's state variables, since by the time the agent is age 20, their parents are age 55, and grandparents are age 90 which means they just passed away. As it would become much clearer below, parents would already have received bequest from the grandparents when they start age 55, so the level of asset or number of children of grandparents no longer matter.
10 We use Rouwenhorst, 1995) method as it is better suited for processes with high persistence. In addition, as the estimates of Storesletten et al. (2004) are for an annual process, we first convert them into values for a 5-year process before discretization.
11 See Tauchen and Hussey (1991).
12 This calibration strategy requires the top three ability levels to be scaled by a factor of 1.35.
13 The estimates of bequest to GDP ratio are wide-ranging. For instance, Benhabib et al. (2019) on the other hand generate a high bequest to GDP ratio of 18.9%. As our calibration targets a bequest to GDP ratio on the low side, our model captures a relatively conservative role of bequests.
14 This social security benefit formula is widely used in the literature to approximate the actual U.S. social security rules (see Hosseini et al. (2021), for instance).
15 We use the Survey of Consumer Finances, since it oversamples wealthy families and has a weighting scheme that corrects for under-coverage at the top of the wealth distribution. This attempts to correct for the outsize role that non-respondents among the very wealthy would play in creating a non-representative sample.
16 The data counterparts are constructed from the 2014HRS data. We computed the average number of children for households in each wealth quintile, and scaled them so that the fertility rate of the 3rd quintile is 2.

References

Aiyagari, S. R. (1994) Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics* 109(3), 659–684.
Altonji, J. G., F. Hayashi and L. J. Kotlikoff. (1992) Is the extended family altruistically linked? Direct tests using micro data. *The American Economic Review* 82, 1177–1198.
Altonji, J. G., F. Hayashi and L. J. Kotlikoff. (1997) Parental altruism and inter vivos transfers: theory and evidence. *Journal of Political Economy* 105(6), 1121–1166.
Alvaredo, F., A. B. Atkinson, T. Piketty and E. Saez. (2013) The top 1 percent in international and historical perspective. *The Journal of Economic Perspectives* 27(3), 3–20.
Bell, F. C., A. H. Wade and S. C. Goss. (1992) *Life Tables for the United States Social Security Area, 1900-2080*. Baltimore, MD: Office of the Actuary, Social Security Administration.

Benhabib, J., A. Bisin and M. Luo. (2019) Wealth distribution and social mobility in the us: A quantitative approach. *American Economic Review* 109(5), 1623–1647.

Bewley, R. (1986) *Allocation Models: Specification Estimation, and Applications*. Cambridge, MA: Ballinger Pub Co.

Cagetti, M. and M. De Nardi. (2006) Entrepreneurship, frictions, and wealth. *Journal of Political Economy* 114(5), 835–870.

Castaneda, A., J. Diaz-Gimenez and J.-V. Rios-Rull. (2003) Accounting for the us earnings and wealth inequality. *Journal of Political Economy* 111(4), 818–857.

Caucutt, E., N. Guner and J. Knowles. (2002) Why do women wait? Matching, wage inequality, and the incentives for fertility delay. *Review of Economic Dynamics* 5(4), 815–855.

Cox, D. (1987) Motives for private income transfers. *Journal of Political Economy* 95(3), 508–546.

Daruich, D. and J. Kozlowski. (2020) Explaining intergenerational mobility: the role of fertility and family transfers. *Review of Economic Dynamics* 36, 220–245.

De La Croix, D. and M. Doepke. (2003) Inequality and growth: why differential fertility matters. *The American Economic Review* 93(4), 1091–1113.

De la Croix, D. and M. Doepke. (2004) Public versus private education when differential fertility matters. *Journal of Development Economics* 73(2), 607–629.

De Nardi, M. (2004) Wealth inequality and intergenerational links. *The Review of Economic Studies* 71(3), 743–768.

De Nardi, M. (2015) *Quantitative models of wealth inequality: A survey*. NBER working paper 21106.

De Nardi, M. and F. Yang. (2016) Wealth inequality, family background, and estate taxation. *Journal of Monetary Economics* 77, 130–145.

Díaz-Giménez, J., A. Glover and J.-V. Ríos-Rull. (2011) Facts on the distributions of earnings, income, and wealth in the united states: 2007 update. *Federal Reserve Bank of Minneapolis Quarterly Review* 34(1), 2–31.

Fuster, L., A. Imrohoroglu and S. Imrohoroglu. (2003) A welfare analysis of social security in a dynastic framework. *International Economic Review* 44(4), 1247–1274.

Fuster, L., A. Imrohoroglu and S. Imrohoroglu. (2007) Elimination of social security in a dynastic framework. *Review of Economic Studies* 74(1), 113–145.

Gale, W. G. and J. K. Scholz. (1994) Intergenerational transfers and the accumulation of wealth. *The Journal of Economic Perspectives* 8(4), 145–160.

Greenwood, J., N. Guner and J. Knowles. (2003) More on marriage, fertility, and the distribution of income. *International Economic Review* 44(3), 827–862.

Hansen, G. D. (1993) The cyclical and secular behaviour of the labour input: comparing efficiency units and hours worked. *Journal of Applied Econometrics* 8(1), 71–80.

Haveman, R. and B. Wolfe. (1995) The determinants of children's attainments: a review of methods and findings. *Journal of economic literature* 33(4), 1829–1878.

Heer, B. (2001) Wealth distribution and optimal inheritance taxation in life-cycle economies with intergenerational transfers. *The Scandinavian Journal of Economics* 103(3), 445–465.

Hosseini, R., K. Kopecky and K. Zhao. (2021). How important is health inequality for lifetime earnings inequality? Unpublished manuscript.

Huggett, M. (1993) The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control* 17(5-6), 953–969.

Hurd, M. D. and J. P. Smith. (1999). Anticipated and actual bequests. Technical report.

Imrohoroglu, A. (1989) Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political Economy* 97(6), 1364–1383.

Imrohoroglu, A. and K. Zhao. (2018) The chinese saving rate: long-term care risks, family insurance, and demographics. *Journal of Monetary Economics* 96, 33–52.

Imrohoroglu, A. and K. Zhao. (2020) Household saving, financial constraints, and the current account in China. *International Economic Review* 61(1), 71–103.

Jones, L. E. and M. Tertilt. (2008) Chapter 5 an economic history of fertility in the united states: 1826-1960. In Rupert, P. (ed.) *Frontiers of family economics*, pp. 165–230, Emerald Group Publishing Limited.

Knowles, J. (1999). Can parental decisions explain us income inequality? Unpublished manuscript, University of Pennsylvania.

Kotlikoff, L. J. and L. H. Summers. (1981) The role of intergenerational transfers in aggregate capital accumulation. *Journal of Political Economy* 89(4), 706–732.

Krueger, D., K. Mitman and F. Perri. (2016) Macroeconomics and household heterogeneity, In: *Handbook of Macroeconomics*, vol. 2, pp. 843–921, Elsevier.

Krusell, P. and A. A. Smith, Jr.. (1998) Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy* 106(5), 867–896.

Lee, S. Y. T., N. Roys and A. Seshadri (2024). The causal effect of parents' education on children's earnings. Technical report.

Rouwenhorst, K. G. (1995) *Asset Pricing Implications of Equilibrium Business Cycle Models, vol. 6 of Frontiers of Business Cycle Research*. Princeton, NJ: Princeton University Press.

Sommer, K. (2016) Fertility choice in a life cycle model with idiosyncratic uninsurable earnings risk. *Journal of Monetary Economics* 83, 27–38.

Storesletten, K., C. I. Telmer and A. Yaron. (2004) Consumption and risk sharing over the life cycle. *Journal of monetary Economics* 51(3), 609–633.

Suen, R. M. (2014) Time preference and the distributions of wealth and income. *Economic Inquiry* 52(1), 364–381.

Tauchen, G. and R. Hussey. (1991) Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica* 59(2), 371.

Wang, K. (2016). Bequests in the us: Patterns, motives, and tax policy. PhD Thesis, TAMU.

Wilhelm, M. O. (1996) Bequest behavior and the effect of heirs' earnings: Testing the altruistic model of bequests. *The American Economic Review* 86, 874–892.

Zhao, K. (2011) Social security, differential fertility, and the dynamics of the earnings distribution. *The B.E. Journal of Macroeconomics (Contributions)* 11(1), 1–31.

Zhao, K. (2014) War finance and the baby boom. *Review of Economic Dynamics* 17(3), 459–473.

Zimmerman, D. J. (1992) Regression toward mediocrity in economic stature. *The American Economic Review*, 82(3), 409–429.

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