PHOTON AND MAGNETIC ALIGNMENT OF INTERSTELLAR DUST GRAINS

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Abstract. We have considered the cooperative effects of photon and magnetic alignment on plausible models for interstellar dust grains. Neither effect, either in cooperation or alone, appears able to produce the observed alignment.

Previous papers (Harwit, 1970a, b; Purcell and Spitzer, 1971; Jones and Spitzer, 1967; Purcell, 1969) have examined various mechanisms to account for the observed alignment of interstellar dust grains. We here examine in more detail the effects of absorption of interstellar starlight photons and re-emission of infrared radiation on the most often mentioned alignment mechanism, paramagnetic relaxation.

Let us take as a model for a grain a cylinder of radius r and length $\ell = \eta r$. Consider the angular momentum of the grain to be statistically in equilibrium, the various additions and substractions of angular momentum being equal. Take the X-Y plane to be the galactic plane with 95% of the starlight photon flux lying in the plane; assume that the gas flow around the grain is isotropic and that the magnetic field B is in the Y direction.

Consider now the various contributions to the angular momentum squared of the grain. Tending to add angular momentum in a random walk fashion are ΔL_S^2, the incremental angular momentum squared carried by starlight photons, ΔL_R^2, that carried by isotropically thermally reradiated photons, ΔL_T^2, that carried by anisotropically thermally reradiated photons, the anisotropy being due to the alignment of the grain, and ΔL_G^2, that contributed by isotropic gas collisions with the grain. Tending to remove angular momentum are ΔL_D^2, the damping due to friction with the interstellar gas, ΔL_m^2, the damping due to paramagnetic relaxation, and ΔL_P^2, the damping due to more probable emission of photons which remove angular momentum than those that add. While the first 4 terms are random walk processes, the last 3 are systematic.

Let us take as the alignment the excess angular momentum in the X and Y directions as compared to an isotropically rotating grain. Thus:

$$Q_j = \frac{L_X^2 + L_Y^2}{L_X^2 + L_Y^2 + L_Z^2} - \tfrac{2}{3},$$

where L_i^2 is the rotational angular momentum squared in the ith direction. To find these we must solve the 3 equations:

$$\sum_j \Delta L_{ji}^2 = 0,$$

Greenberg and Van de Hulst (eds.), Interstellar Dust and Related Topics, 197–202.

where $j = S, R, T, G, D, M, P$ and $i = X, Y, Z$. These must be solved by a self consistent calculation as the moment of inertia of the grain in each direction depends on the degree of alignment. We take $\Delta L_{mP} = 0$, as the magnetic field is in the Y-direction.

The various terms are:

Photon Angular Momentum Contributions

Starlight

$$\Delta L^2_{Si} = N_i(r,\eta)\, c q \hbar^2 r^2 \pi\, (\eta+1)\left(\frac{1}{3} + F(\eta, d_i)\right)\tau$$

Isotropic Infrared Re-Emission

$$\Delta L^2_{Ri} = \left(\sum_{i=X}^{Z} \Delta L^2_{Si}\right)\beta/3q$$

Anisotropic Infrared Re-Emission

$$\Delta L^2_{Ti} = F(\eta, d_i)\, \Delta L^2_{Ri}$$

Photon Damping

$$\Delta L^2_{Pi} = -I(d_i)\,\omega_i^2\, k' \frac{\left(\sum_{i=X}^{Z} N_i(r,\eta)\right)}{3} C\beta\hbar^2\, r^2 \pi \frac{(\eta+1)}{3}\,\tau$$

Fractional Excess Area Exposed

$$F(\eta, d_i) = \frac{\eta\sqrt{\eta^2+4} + 1 + d_i(\eta\sqrt{\eta^2+4} - 1) - (2\eta\sqrt{\eta^2+4} + 2)/3}{\dfrac{2\eta\sqrt{\eta^2+4} + 2}{3}}$$

Gas and Magnetic Angular Momentum Contributions

Isotropic Gas Collisions

$$\Delta L^2_{Gi} = nv \frac{(mvr)^2}{3} \frac{r^2}{4}\left(\pi + \frac{\eta^3}{3}\right)\tau$$

Gas Frictional Damping

$$\Delta L^2_{Di} = -I(d_i)\,\omega_i^2 \frac{nvm}{\sqrt{3}} \frac{V^{4/3}}{C}\,\tau$$

Paramagnetic Relaxation Damping

$$\Delta L^2_{Mi} = -I(d_i)\,\omega_i^2\, KVB^2\left(\frac{T_{Gas} - T_{Grain}}{T_{Gas} + T_{Grain}}\right)\tau$$

Moment of Inertia

$$I(d_i) = V\rho\frac{r^2}{4}\left(1 + \frac{\eta^2}{3}\right) d_i + \frac{1-d_i}{2}\left[V\rho\frac{r^2}{4}\left(1 + \frac{\eta^2}{3}\right) + V\rho\frac{r^2}{2}\right]$$

$$d_i = \frac{\omega_i^2}{\omega_X^2 + \omega_Y^2 + \omega_Z^2}$$

$F(\eta, d_i)$ is a result of the partial alignment of the grain. $N_i(r, \eta)$ is the number of photons absorbed by the grain and is obtained by integrating a suitable efficiency factor over an approximate interstellar radiation field. We take this factor to be $(\nu/\nu_0)^2$ below ν_0 and 1 above, where $\nu_0 = c/2\pi r\sqrt{\eta}$ (Purcell and Spitzer, 1971). The radiation field used consists of the superposition of 3 blackbody spectra, corresponding to temperatures of 14500 K, 7500 K and 4000 K multiplied by dilution factors of 4×10^{-16}, 1.5×10^{-14}, and 1.5×10^{-13} respectively (Werner and Salpeter, 1969). The terms ΔL_{Di}^2 and ΔL_{mi}^2 are taken from Purcell and Spitzer (1971) by integrating the expressions they give for the damping torques on the grain. The factor $3^{-1/2}$ comes from having to use the average angle at which the gas molecules strike the grain; the factor $(T_{\text{gas}} - T_{\text{grain}})/T_{\text{gas}} + T_{\text{grain}})$ is inserted to take into account the temperature dependence.

q is the correlation between absorbed and scattered photons, the forward scattered ones having no effect on the angular momentum while the backward scattered ones contribute to the random walk process as do the absorbed ones. We estimate that $q = 1.2$ is reasonable. β is the number of thermalized infrared photons emitted for each starlight photon absorbed. Estimating the grain temperature to be 30 K we divide the average interstellar photon temperature by this and thus obtain $\beta = 143$. k' is a number that determines the excess number of emitted photons which remove angular momentum. From Harwit's paper we have:

$$\frac{\text{Probability}(-)}{\text{Probability}(+)} = \frac{(\omega_p + \omega)^3}{(\omega_p - \omega)^3} \frac{2J+1}{2J} \cong 1 + \frac{3\omega}{\omega_p}.$$

Thus $k' = 3/\omega_p$.

To fix numerical values we choose gas characteristics $v = 1.7 \times 10^5$ cm s^{-1}, $T_{\text{gas}} =$ 120 K, and $m = 1.6 \times 10^{-24}$ gm: the grain density ρ we take to be 1 gm cm^{-3}; B we take to be 3×10^{-6} G; the frequency of the emitted radiation, ω_p, is 3×10^{12} s^{-1}; V is the volume of the grain and K as defined in Spitzer and Jones (1967) we take to be about 10^{-13}. n is the number of gas molecules per cubic centimeter.

The main results of the calculation are presented in Tables 1 and 2. Table 1 presents a comparison of the amount of alignment obtained with gas, magnetic, and photon effects operating, vs the alignment obtained with just gas and magnetic effects as a function or the radius and the eccentricity of the grain and the number of gas molecules we assume per cubic centimeter. The last line presents the result of averaging over the 5 values of r and eccentricities from 1 to 10. It can be seen that the net photon effects are least for the larger grains and for higher values of the gas density, the effect being one of lessening the alignment for the small grains. As it is the smaller grains that are best aligned by paramagnetic relaxation (Purcell and Spitzer, 1971), the net photon effect is to increase somewhat the magnetic field needed to account for the alignment. It is seen that the net disaligning effect is smaller than that estimated by Purcell and Spitzer (1971) for spherical grains, especially for the larger values of r.

Table 2 presents the ratio of photon and magnetic alignment when they are not working in conjunction. It can be seen that the photon aligning mechanism gives

TABLE 1

$\frac{\text{PHOTON AND MAGNETIC ALIGNMENT}}{\text{MAGNETIC ALIGNMENT}}$

EC = ECCENTRICITY = $\eta/2$ R = RADIUS

NUMBER OF GAS MOLECULES/CM^3

		0.1	1.0	3.0	10.0
R = 3.16 x 10^{-7}CM	EC=2	.273	.337	.457	.678
	EC=5	.166	.331	.550	.810
R = 1.00 x 10^{-6}CM	EC=2	.113	.220	.399	.693
	EC=5	.0886	.328	.607	.911
R = 3.16 x 10^{-6}CM	EC=2	.0725	.252	.508	.858
	EC=5	.110	.527	.862	1.12
R = 1.00 x 10^{-5}CM	EC=2	.128	.542	.854	1.08
	EC=5	.321	.909	1.07	1.16
R = 3.16 x 10^{-5}CM	EC=2	.443	.935	1.03	1.08
	EC=5	.794	1.03	1.06	1.06
AVERAGE OVER R AND EC		.305	.627	.825	1.01

TABLE 2

$\frac{\text{PHOTON ALIGNMENT}}{\text{MAGNETIC ALIGNMENT}}$

EC = ECCENTRICITY = $\eta/2$ R = RADIUS

NUMBER OF GAS MOLECULES/CM^3

		0.1	1.0	3.0	10.0
R = 3.16 x 10^{-7}CM	EC=2	.0101	.0123	.0164	.0237
	EC=5	.0117	.0225	.0354	.0490
R = 1.00 x 10^{-6}CM	EC=2	.0110	.0212	.0385	.0672
	EC=5	.0157	.0562	.100	.144
R = 3.16 x 10^{-6}CM	EC=2	.0138	.0482	.0977	.166
	EC=5	.0286	.131	.207	.262
R = 1.00 x 10^{-5}CM	EC=2	.0228	.0971	.154	.196
	EC=5	.0567	.148	.171	.183
R = 1.00 x 10^{-5}CM	EC=2	.0378	.0797	.0881	.0921
	EC=5	.0533	.0648	.0690	.0685
AVERAGE OVER R AND EC		.0353	.0878	.118	.140
DEVIATION FROM PURCELL AND SPITZER METHOD		16.9%	8.8%	6.3%	6.8%

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alignments far smaller than that given by magnetic effects, which already yields Q_j's which are too small. The difference in the Q_j's obtained here from those of previous papers (Harwit, 1970a, b) comes from taking a larger value of β as indicated from infrared rocket observations (Houck *et al.*, 1971) and considering the systematic disalignment more rigorously. These considerations indicate that photon alignment is not sufficient to produce the observed polarization.

The last line of the table shows a comparison between this method of calculation and that employed by Purcell and Spitzer (1971) for the case in which we have no photon effects. The values given are the average over r and eccentricity of the absolute value of the difference in alignment obtained by the two methods divided by the sum of the alignments. It was found that the agreement is reasonably close in all cases, the best agreement being in the cases of higher gas density and radii between 3.16×10^{-6} cm and 3.16×10^{-5} cm, conditions which are more probable than the other ones treated.

From this calculation we therefore conclude that even taking into account the effect of instantaneous alignment of the grain on the various angular momentum contributions, neither paramagnetic relaxation nor photon alignment are sufficient to produce the observed alignment, either alone or working in conjunction with each other.

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TABLE OF SYMBOLS

n = Number of Gas Molecules/cm^3

v = Velocity of Gas Molecules

m = Mass of Gas Molecules

ρ = Density of the Grain

V = Volume of the Grain

r = Radius of Grain

η = Ratio Length of Grain to Radius

ω = 2π Frequency of Rotation of Grain

$q = 1 + \dfrac{\text{Number of Backward Scattered Photons}}{\text{Number of Absorbed Photons}}$

β = Number of IR Photons Emitted for Each Photon Absorbed

$N_i(r, \eta)$ = Integrated Number of Photons Absorbed by the Grain

k' = 3/Frequency of IR Photons Emitted

$K = \chi''/\omega$ = Imaginary Part of Volume Susceptibility $/\omega$

C = Factor Depending on Shape of the Grain

B = Interstellar Magnetic Field Strength

T = Temperature

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