

Index

- A-D-E 169–175, 204–205, 295, 373–374, 387, 389, 395, 430
- absolute invariant (*see* Hauptmodul)
- action (physics) 229–230, 250, 259–260, 278
- adèle 157–158
- adjoint
 - matrix 66
 - operator 47, 49, 52, 256
 - operator $\text{ad } x$ (Lie algebra) 60, 190
 - representation 57, 60, 67–68, 72–73, 193, 331–332
 - module (*see* representation)
- Ado–Iwasawa Theorem 58
- affine algebra 176
 - and braid groups (*see* braid group \mathfrak{B}_n and KZ equation)
 - characters (*see* character, for an affine algebra)
 - graded dimensions (*see* character, for an affine algebra)
 - and Lie algebras of polynomial growth 188–189
 - and loop algebras (*see* loop algebra)
 - and loop groups (*see* loop group)
 - modules 189, 192, 208, 329
 - integrable highest-weight 194–197, 199, 205, 215, 286, 329, 333
 - nontwisted
 - construction 189–191, 205, 312
 - in string theory 189, 280, 286–288, 372–373
 - twisted 375
 - construction 191–192
 - and twisted character 220, 346–347
 - and Virasoro/Witt algebras (*see* Virasoro algebra; Witt algebra)
 - Weyl group (*see* Weyl group)
 - (*see also* Kac–Moody algebra)
- Alexander’s theorem 43
- algebra 8, 14, 24, 29, 44, 53, 98, 412, 431
- algebraic closure 63, 395
- algebraic group 58
- algebraic integer 97, 394
- algebraic number 96–97
- alternating group \mathcal{A}_n 19, 100, 383
- amplitude (quantum) 250, 252, 259, 276, 279–280, 300, 308
- annihilation operator \hat{a} 248, 257–258
- anomaly 44, 134, 166, 259
 - conformal 134, 181, 259, 285, 318
 - global 178
 - local 178
- anti-associativity 53
- anti-commutativity 53
- anti-holomorphic 281–282, 285
- anti-particle 255, 258, 360
- arrow (category) 87, 305
- associativity constraint 89–90, 398
- associator 398–399
- asymptotic inclusion 390
- Atiyah–Bott fixed-point theorem 149–150
- Atiyah–Singer Index Theorem 44, 149, 181, 204, 270, 271, 350
- Atkin–Lehner involution 408, 427
- automorphic form/function 104, 137, 146, 154–158, 166, 223–225
- automorphism
 - of groups 15
 - inner 70–71, 79, 346
 - of lattices 32, 43, 153, 337, 409
 - of Lie algebras 191, 206, 218–220
 - outer 79
 - and representations 80–81, 218
 - of surfaces 115–116, 121, 124
 - of VOAs 325, 329, 331, 338, 343, 346, 382–383, 424
 - of von Neumann algebras 386
 - (*see also* Galois, group)
- Baby Monster \mathbb{B} 404, 422
 - Lie algebra 416
 - moonshine 423–424
 - vertex operator superalgebra 423
- Baker–Campbell–Hausdorff formula 58, 162
- base of root system 70
- bilinear form 307, 325, 329
 - invariant
 - for Lie algebras 60, 208, 209–211, 213, 324; (*see also* Killing form)
 - for VOAs 321–322, 331, 332
- bilinear product 53–54, 318
- bimodule 388–390
- Bimonster $\mathbb{M} \wr \mathbb{Z}_2$ 402, 406, 433

- Bloch group 372
- Borcherds' Conjecture 430
- Borcherds' identity 320
- Borcherds lift 223–225
- Borcherds–Kac–Moody algebra
- basic theory 212–214, 217–218, 220, 222
 - and Monstrous Moonshine 222, 415–416; (see also Monster Lie algebra)
 - in physics 205, 214, 225, 375
 - (see also Baby Monster, Lie algebra; fake Monster Lie algebra; Monster Lie algebra)
- Borel subgroup 59, 83–84
- Borel–Weil Theorem 83–84, 86, 184–186, 207
- boson 168, 250, 257, 277, 295–296, 311, 352, 360, 371
- bounded operator 46–47, 49, 52, 82, 275–276
- bracket $[x, y]$ 53, 54
- braid group \mathcal{B}_3
- as mapping class group 125, 420–421
 - and Moonshine 11, 166, 288, 421–422, 435
 - and quilts 421, 425, 427
 - and $SL_2(\mathbb{Z})$ 11, 27, 165, 202, 421, 427
 - and trefoil 164
- braid group \mathcal{B}_n
- Burau representation of 27–28, 165
 - centre 27
 - definition 26
 - degree 27, 165
 - and free group 27, 396, 401
 - as fundamental group 27, 40–41, 202, 291, 296, 396
 - and KZ equation 201–202
 - and links 43, 385
 - and mapping class groups 121–122
 - presentation 54
 - pure \mathcal{P}_n 27
 - representations of 27–29, 166–167, 296, 378, 384–385, 388
 - and symmetric group 27, 41, 296
- braid statistics 250, 296–297, 422
- braiding operator (CFT) 202, 364–366, 381, 398
- Burau representation (see braid group \mathcal{B}_3 ; braid group \mathcal{B}_n)
- $c/24$ 133, 140, 168–169, 181, 187, 198–199, 321, 339–342, 366
- C^∞ (smooth) 33, 45
- C_2 -cofinite condition 336–338, 342–344, 348, 375
- Calabi–Yau manifold 214, 351–352, 395, 432–433
- canonical commutation relations 54, 238, 248, 256
- Cartan matrix
- $Cartan_{BK M}$ 212–213, 415
 - $Cartan_{KM}$ 209–211
 - $Cartan_{ss}$ 31, 62–63, 68–70, 169
- Cartan subalgebra
- for affine algebras 190–191, 194, 205, 207
 - for Borcherds–Kac–Moody algebras 213, 415
 - for Kac–Moody algebras 210, 211
 - for semi-simple Lie algebras 69–70, 78, 79, 81, 84, 183, 204
- category
- and Borel–Weil 185
 - Braid** of braids 87, 89, 91–92, 385
 - braided monoidal 90–93, 202, 331, 399
 - and braids 93, 398 - definition 87
 - modular 94, 202, 376, 385–386, 388, 400
 - monoidal (or tensor) 89–90, 265
 - quotient 87
 - ribbon 93, 202, 384, 399–400
 - and link invariants 94 - Ribbon** of ribbons 93–94, 384, 400
 - Riem** of Riemann spaces 87–88, 302
 - strict 90
 - symmetric monoidal 90, 202, 331
 - tensor (see monoidal above)
 - Vect** of vector spaces 87, 89–90, 302, 305
- Cayley's Theorem 58
- central charge c
- and affine algebra modules 195, 198, 199
 - and CFT 259, 284–286, 288–289, 291, 299
 - meaning of 198, 259, 341
 - and Virasoro modules 182, 199, 284, 318
 - and VOA modules 318, 328, 341
 - (see also $c/24$)
- central extension
- of group 178–179, 186
 - of Lie algebra 179–180, 181, 186
 - and projective representations 165, 178–179, 181, 186, 345
- centraliser
- in group 295, 382, 403
 - in Lie algebra 73
 - in vertex operator algebra 328
- centre
- of von Neumann algebra 50
 - $Z(G)$ of group 17, 19, 59
 - $Z(\mathfrak{g})$ of Lie algebra 59, 415
- centre construction 380
- character
- for an affine algebra 7, 137, 144, 192–196, 288, 337–339, 368, 372
 - modularity (explanation) 74, 194–196, 200, 202, 205, 280, 288, 339–342
 - modularity (statement) 176, 196–197, 199–200, 288, 342, 344, 352–353
- Brauer 429–430, 434
- for groups 5, 23–25, 101, 304–305, 382, 389
 - Jacobi (for VOAs) 338–339, 344–345

- character (*cont.*)
 ring 24, 356–358
 for semi-simple Lie algebras 76–78, 368
 table 24, 356, 358, 367, 391
 twining (*see* character, twisted)
 twisted 80–81
 virtual 412, 423
 for a VOA 321, 337–338, 420
 modularity 338–343, 420–421
- characteristic 15, 25–26, 63, 428
- charge-conjugation 302, 332, 356, 358, 360–361, 363, 386
- Chevalley group (*see* finite group of Lie type)
- Chevalley involution 67, 211
- chiral algebra (*see* vertex operator algebra)
- chiral block (*see also* mapping class group)
 285–288, 290–291, 297, 308, 376, 392
 and correlation functions 285–287, 291, 298
 and graded dimensions 287–288, 341–342, 420–421
 and KZ equation 150, 186–187, 201–202, 288, 290–292
 and moduli space 287, 290–291
 spaces of 287–288, 291–292, 302, 310, 357, 363–366, 400
- chiral de Rham complex \mathcal{MSV}_X 351–353
- chiral primaries (*see* primaries; state)
- class function 23, 81, 429
- Clebsch–Gordon coefficients (or 3j-symbols) 365
- cobordism 87, 305, 352
- cocycle 382, 426
 2-cocycle 177–178, 180, 186, 215, 346–348
- coherence (categories) 89–92
- cohomology
 BRST 325, 329, 416
 elliptic 432
 group 20, 177, 179
 Lie algebra 180
 Moonshine 416
 and Nakajima’s affine algebra construction 205
 quantum (*see* Gromov–Witten invariants)
 semi-infinite 416
 sheaf 352
- co-labels a_i^\vee 190–192, 193–195
- commensurable 128, 291–292
- commutant 49
- commutativity constraint 90–92, 398
- compact support 45, 314
- complex multiplication 392–395, 401
- composition factor 18–19, 24, 28, 429
- composition series 18, 75
- configuration space 202, 229–231, 237, 291, 296
- conformal block (*see* chiral block)
- conformal equivalence 114–115, 279, 282
- conformal field theory (CFT) 253, 280–281, 301
 boundary 289, 307, 374, 433
 at $c \leq 1$ 288–289, 295
 characters (*see* graded dimension, and RCFT)
 and Galois 358, 360, 362, 371, 395, 397–401, 421
 heterotic 294, 433
 and moduli space (*see* moduli space, in CFT)
 and Monster 294, 303, 433–434
 and Moonshine 9–10, 226–227, 285, 292, 433–434
 nonunitary 282, 359, 376
 quasi-rational 197, 343, 376–377
 rational (RCFT) 197, 226, 286–287, 302–303, 310
 Segal’s axioms (*see* Segal’s axioms)
 and strings (*see* string theory)
 super 301, 351–353, 374, 395
 and two dimensions 280–282, 297
 (*see also* chiral block; vertex operator algebra)
- conformal group 281–282, 285
- conformal map 107
 and holomorphicity 114, 281–282
- conformal primary (*see* primaries)
- conformal vector ω 318, 326, 377
 (*see also* stress-energy tensor)
- conformal weight 182, 194–195, 199, 284–286, 288, 318, 322, 421
- congruence property 196, 360–361, 400
- congruence subgroup 128, 138–139, 146–147, 166;
 (*see also* congruence property;
 moonshine-type)
- conjugacy class K_g 23, 77, 305, 375, 402, 411
- connection 38, 178, 201–202, 269, 389
- conserved charge 231–232, 240, 259, 268, 284, 373
- conserved current 238, 240, 259, 284, 372–373
- convolution product 21, 84–85, 136, 148, 377
- Conway groups 32, 177, 179, 404, 409, 414–415, 422–424
- Conway–Norton conjectures (*see* Monstrous Moonshine, conjectures)
- co-root 195, 211
- correlation function 252–253, 259–260, 283
 in CFT 279, 281, 283–287, 291–292, 298, 307, 333, 361, 363
 (*see also* chiral block)
- coset construction 328
- cotangent bundle 37, 231, 232
- covariant derivative 38, 178
- covering transformation 100
 (*see also* geometric Galois)
- Coxeter–Dynkin diagram 31, 62–64, 70–71, 79, 169, 368
 affine (*see* extended *below*)
 extended 62, 170, 172, 190–192, 218–219, 368, 370, 373–374, 431
- creation operator \hat{a}^\dagger 193–195, 248, 257–258
- critical point (*see* singularity)

- crossed-product $M \rtimes G$ 50–51, 386, 389–390
- crystallographic condition 188, 214
- current algebra 189
- curvature 106, 108, 114–115, 122, 178, 201, 269, 285
- curve (1–dimensional manifold)
 complex 1–2, 110, 112, 114, 122, 280
 real 35–36, 38, 40, 280
- cuspidal 2, 127–128, 130–131, 164, 223–224, 280, 291–292, 427
 (see also Deligne–Mumford compactification; surface with nodes)
- Cvitanović’s Magic Triangle 72–73
- cyclotomic
 character χ^{cyclo} 397
 field 101–102, 358, 418, 419
 integers 102, 425, 427, 429–430
 and characters 101, 103
 and modular functions 140, 147
- D-brane 278, 280, 292, 374, 400, 433
- Dedekind eta function $\eta(\tau)$ 11, 44, 131–133, 140, 165, 270, 409
 in Lie theory 183, 193
 in physics 133, 371
- Dedekind sum 132, 165
- degree of braid (see braid group)
- degree of field extension 96, 100, 102
- Dehn twists 120, 122, 125, 420–421
- Deligne–Mumford compactification $\overline{\mathcal{M}}_{g,n}$ 123–124
 (see also cusp; surface with nodes)
- denominator identity 131, 176, 220–223, 225, 415–416
- derivation 36, 55, 61, 190, 320
 adjoined to Lie algebras 188, 190, 213, 215
- $\text{Diff}^+(S^1)$ 64, 184, 186, 206–207, 282, 299–300
 complexification? 184, 300–301
 representations 180, 184–185
 universal cover 184
 (see also Virasoro algebra; Witt algebra)
- diffeomorphism 34, 36, 57, 306, 432
 of surface 111, 285, 299
- differential form 37, 119, 180
- differentials 352
- dihedral group \mathcal{D}_n 16, 17, 20, 24, 28, 170, 178, 187–188
- dilogarithm 371–372
- Dirac delta $\delta(x)$ 21, 46–47, 243, 265, 313–316
- direct integral 48–49, 50, 286
- direct limit (see injective limit)
- direct product 16, 18, 35, 59, 88, 95
- direct sum
 and categories 88, 94, 95
 and direct integral 50, 85
 of eigenspaces 75, 78
 of lattices 31, 70
 of Lie algebras 59–61, 63, 180
 of modules (see modules)
- Dirichlet character 146–147, 158
- Dirichlet series 104, 141–142
- discrete series (of Virasoro modules) 182–185, 194, 216, 288
- distribution 46–47, 160–162, 265, 266, 282–283, 313
 tempered 46–48, 243, 253, 272, 274
- \mathcal{D} -module 185, 189, 203
- Drinfel’d double (see quantum double)
- dual Coxeter number h^\vee 73, 195, 326–327, 368, 371, 373–374
 shift by (see h^\vee -shift)
- duality (in CFT) 289, 301, 320, 322, 365–366
- Dynkin diagram (see Coxeter–Dynkin diagram)
- E_8 336, 353, 369, 381, 389
 lattice 31–32, 70, 138, 409
 Lie algebra 61, 73, 77, 78, 196–197, 414, 431
 Lie group 196
 and Moonshine 6–7, 9, 196–197, 412
 Weyl group 406, 409
 (see also A–D–E)
- Eisenstein series 2–3, 8, 112, 127–128, 134, 152, 156, 164
- electromagnetism 233–239, 255, 260, 267–269, 277
- elliptic curve 56, 117, 129–130, 142, 401, 432
- elliptic functions 117–118, 129, 143, 159, 203
- elliptic genus (see genus, elliptic)
- energy 229, 232, 235, 239, 254
 and conformal weights 181–182, 207, 322
 conservation 228, 229–230, 232
 kinetic 229
 operator (see Hamiltonian)
 potential (see potential)
- energy-momentum vector 235, 254, 272
- enhanced mapping class group (see mapping class group, enhanced)
- equal-time commutation relations 255, 263, 265–266, 271
- equations of motion 227, 229–230, 232, 235, 242, 247, 254
- Erlangen Programm 38
- Esquisse d’un Programme* 397–401
- exponents (Lie theory) 373–374
- factor (von Neumann algebras)
 classification 50
 crossed-product construction of (see crossed-product)
 definition 49–50
 and Galois analogy 100, 386
 hyperfinite 50, 386–387
 modules of 52
 and projections 51
 in quantum field theory 50

- factorisation (CFT) 280, 291
 fake Monster Lie algebra 212, 214, 225, 416
 Feit–Thompson Theorem 18
 Fermat curve F_n 393–395, 401
 Fermat’s Last Theorem 97, 101, 112, 130, 142
 fermion 250, 255, 258, 296
 in CFT 168, 281, 301, 328, 352, 371, 423
 Feynman diagram 251–253, 260–262, 265, 271
 in string theory 278–279, 281
 Feynman path formulation 250–251, 259–260, 269,
 296, 300, 348, 420–421
 Feynman rules 260–261, 364
 fibre 38
 fibre bundle 38, 269
 field (in algebra and number theory)
 base 96
 cyclotomic (*see* cyclotomic, field)
 definition 15
 extension 96
 finite extension 96, 99
 Galois (*see* Galois, extension)
 finite \mathbb{F}_q 15, 19, 25–26, 28, 79, 142, 371, 424,
 429–430
 field equation (physics) 238, 255–256, 258
 Fields medal 11, 43, 47, 49, 137, 248, 277,
 413
 finite group of Lie type 19, 382, 424
 finite simple group
 classification 4, 19–20, 426
 definition 17–18
 importance 18–20
 and Lie theory 19, 79
 and representations 24, 103
 sporadic 19–20, 32, 402–404, 406, 422–425, 430
 (*see also* Baby Monster; Conway groups; Fischer
 groups; Mathieu groups; Monster)
 Fischer groups 404, 431
 Fock space 181, 198, 199, 309
 force 227–229, 236, 267–269, 277
 formal series 313–317, 399
 and distributions 47, 313–317
 Fourier
 analysis 21, 147, 207
 coefficient 131, 257
 transform 47, 49, 84–85, 131, 135–137, 256–257,
 359, 377
 fractional linear transformation (*see* Möbius
 transformations)
 free group \mathcal{F}_n 16, 27, 29, 40, 51–52, 114, 128, 203,
 396–399
 Freudenthal’s Magic Square 60–61, 72
 Fricke element (in \mathbb{M}) 425, 428
 Fricke involution 425
 Frobenius algebra 286, 289, 307
 Fuchsian group 109–110, 128, 130, 137
 functional 46–47, 50, 66, 68, 73, 93
 functor 87, 89, 92, 94, 99, 298, 305, 384–385, 416
 modular (*see* modular functor)
 fundamental domain 109–110, 138
 fundamental group $\pi_1(M)$ 27, 40–41, 100, 113–115,
 200–202, 396–397, 425
 algebraic 396
 and Lie theory 59, 178–179
 fundamental groupoid 87, 397
 fusing operator 365, 366
 fusion
 for affine algebras 333, 355, 369–371
 coefficients (*see* multiplicities *below*)
 matrix 356
 multiplicities 95, 331, 336, 356
 product 331, 337
 ring 331, 355–360
 Galilei group 179, 275
 Galois
 extension 99–100, 147
 group 99, 101–102, 147, 418–419
 absolute group 158, 395–400, 421
 theory 14, 18, 57–58, 99, 100, 150, 158, 347, 386
 Gamma function $\Gamma(s)$ 141
 gauge symmetry 266–269, 383
 Gauss sum 101, 401
 generalisations 154–157, 187, 208–209, 424
 generalised Kac–Moody algebra (*see*
 Borcherds–Kac–Moody algebra)
 generalised Moonshine (*see* Maxi-Moonshine)
 generalised Riemann existence theorem 395–396
 genus
 elliptic 351–353, 431–432
 of surface 110–114, 139, 392
 Witten 353, 432
 geometric Galois 100, 396
 geometric Langlands 327
 geometric quantisation 85, 184
 ghosts 184, 262, 300, 352, 416
 graded character 407, 412, 420, 430
 (*see also* McKay–Thompson series)
 graded dimension
 and affine algebras 7, 192–193
 linear dependence of 332, 337
 and RCFT 10, 287–288, 301–302, 341–342, 392,
 420
 modularity of 10, 288, 291, 301–302, 339–342,
 420–421
 and Virasoro algebra 181–183
 and VOAs 9, 321, 331
 (*see also* character; chiral block)
 Gram matrix (lattice) 30–31
 graph
 Coxeter 188
 dual principal 389
 principal 389

- Grassmannian, infinite 207
 Grassmannian $\text{Gr}(m, n)$ 370
 Griess algebra 325, 329, 403, 414, 423, 430
 Gromov–Witten invariants 265, 370
 Grothendieck–Teichmüller group 399
- group
 abelian 17–18, 29, 31, 85, 88, 95, 102, 357
 action 20–21, 27–28, 32, 68
 algebra $\mathbb{C}G$ 21, 23, 25, 64, 74–75, 334–335
 (see also $L^2(G)$)
 central extension of (see central extension, of group)
 classification of finite 17–18
 Coxeter 187–188, 214, 404, 411
 cyclic \mathbb{Z}_n 15, 17–19, 100
 direct product of (see direct product)
 exponent 407
 extension 17–20, 28, 56, 100
 fabulous 404, 406
 finitely generated 16–17
 homomorphism 15–16, 18, 20, 88
 k -transposition 426–427, 431
 knot 164, 384
 presentation 16, 179
 profinite 395
 reflection 62, 187
 semi-direct product of (see semi-direct product)
 simple 17–18, 79
 (see also finite simple group)
 solvable 18, 24, 85–86, 100
 (see also finite simple group; Lie group)
 group determinant 25
 group representation (see module for group)
- h^\vee -shift (affine algebras) 199, 371
 Haag’s Theorem 265–266
 Haar measure 83, 85, 136
 Hamiltonian 232, 242, 247–248, 254, 257–258, 306, 420
 in CFT 181, 197, 282, 287, 341, 420–421
 harmonic oscillator 228, 230–233, 238, 246, 248–249, 255–257, 276
- Hauptmodul
 classification 139–140, 408
 definition 5, 139
 examples 6, 139
 and modular equations 418
 and replicable functions 411
 (see also j -function; McKay–Thompson series; replicable functions)
- heat equation 147
 heat kernel 148–149
 and KZ equation 11, 150
 and modularity 11, 104, 148–150
 Hecke operator 222, 410
- Heisenberg algebra \mathfrak{heis} 54, 179–180, 194–195, 212, 214, 216, 265, 326
 Heisenberg group 85–86, 104, 155, 159–164, 179
 Heisenberg VOA (see vertex operator algebra, Heisenberg)
 Hermitian form 22, 25, 45, 67, 282, 300
 hexagon axiom 90–91, 398–399
 highest weight (see weight)
 Hilbert space
 definition 45
 in quantum field theory 177, 253, 272, 275
 in quantum mechanics 177, 241–242, 248, 249
 separable 46
 Hilbert’s Problems 56
 Hirzebruch’s ‘prize question’ 431–432
 holomorphic function 200, 266, 274, 281–282
 homeomorphism 33, 41, 100, 110, 306
 homogeneous coordinates (projective space) 39, 108, 112
 homogeneous extension (see direct product)
 homogeneous space G/K 84, 154–155, 184
 Hopf algebra 264, 271, 379–380, 382, 391
 co-commutative 380
 quasi-triangularisable 380, 385
 quasi-triangularisable quasi- 398–399
 Hopf link 385–386, 400
 hyperbolic
 geometry 105–108, 116, 152
 plane 105–106
 reflections 406
 surface 108, 110
- ideal 53, 59–61, 74–75, 98, 102, 429
 idèle 157–158
 index
 of subfactor 100, 386–387
 of subgroup 15
 injective limit 157–158
 inner-product 29, 37–38, 61, 69–70, 238
 instanton 204–206, 265
 intertwining operator 80, 279, 286–287, 331, 342, 363–364, 375–376, 389–390
 inverse Galois problem 395
 inverse limit (see projective limit)
 Ising model 288–290, 366–367
 isogeny 393–394
 isometry 16, 107
 isomorphic groups 15
- j -function $j(\tau)$ 3–4, 6–7, 9, 124, 138, 147, 280, 288, 419
 $J(\tau) = j(\tau) - 744$ 222, 224, 294, 407, 409–410, 415, 417
 Jacobi form/function 143–145, 153, 155, 176, 196, 199, 224, 338, 348
 Jacobi identity 53, 320, 330, 350

- Jacobi triple product identity 221
 Jacobian variety 123, 393–394, 401
 Jones index (*see* index, of subfactor)
 Jones polynomial 42–44, 94, 305, 307, 388
 Jordan–Hölder Theorem 18–19, 60
- K3 surface 205, 432–433
 K-theory 169
 and character rings 370
 and fusion rings 370–371
 of \mathbb{Z} 169
- Kac–Moody algebra
 and affine algebras 176, 210–212
 basic theory 209–211
 hyperbolic (Lorentzian) 210, 224–225, 375
 motivation 187–189
 representation theory 211–212, 223
 (*see also* affine algebra)
- Kac–Walton formula 369
 Kähler class 432
 Killing form $\kappa(x|y)$ 60–61, 64–65, 69–71, 86, 205, 211, 368
- Knizhnik–Zamolodchikov (KZ) connection 150, 201, 292, 399
 Knizhnik–Zamolodchikov (KZ) equation 201–202, 290–291
 monodromy 27, 200, 202, 291, 421
 and Virasoro 185–186, 287, 292, 421
 (*see also* chiral block)
- knot
 ambient isotopic 41
 and braids (*see braid groups and links*)
 crossing number 42
 definition 41
 group (*see* group, knot)
 higher dimensional 41–42
 invariant (*see* link, invariant)
 (*see also* link)
- Krichever–Novikov algebra 216–217
 Kronecker–Weber Theorem 101–102, 158
- $L^2(G)$, $L^2(G/\Gamma)$ etc. 21, 51, 84, 136–137, 155
 labels a_i 170, 172, 190–191, 205, 431
 Lagrangian 229–231, 235, 251, 270
 Lagrangian density 238–240, 255, 257–258, 261, 263–264, 266, 271, 278, 280
 Langlands programme 137, 142, 158
 Laplacian 133, 148–149, 156, 242
- lattice
 automorphisms 32, 43–44, 409
 co-root 195–196
 definition of 6, 29–30
 determinant $|L|$ 31, 328
 dimension 30, 328
 direct sum of (*see* direct sum, of lattices)
 dual L^* 30, 134–135
 equivalent 30
 even 30, 135, 328
 indefinite 30, 214, 224, 328
 integral 30
 laminated 31–32
 positive-definite 30, 328
 root 31–32, 70–71, 169, 188
 self-dual 30, 140, 168, 333, 402
 and string theory (*see string theory and lattices*)
 and tori 32–33, 56
 VOA $\mathcal{V}(L)$ (*see* vertex operator algebra, lattice)
 weight 72–73
 (*see also* Leech lattice, theta series)
- Laurent polynomial 27, 42, 184, 187, 189, 206, 216
 Lax–Phillips scattering theory 154
 Lebesgue integral 46
 Lebesgue measure 45–46
 Leech lattice Λ
 automorphism group Co_0 (*see* Conway groups)
 definition 31–32
 and genus-0 property 428
 and McKay–Thompson series 408–409
 and Moonshine module (*see* Moonshine module, construction)
 theta series 7, 135, 294, 415
 uniqueness 32, 414–415
- level
 of affine algebra module 192–194, 196, 207, 217, 286, 326, 333
 critical 327, 331
 fractional 375
- Levi decomposition 60
 L-function 142, 157, 158, 394
- Lie algebra
 abelian 54, 69, 86
 automorphisms 78–80
 classification 54
 definition 7, 53
 free 212, 416
 geometric 349
 homomorphism 54, 59, 64, 66
 ideal of (*see* ideal)
 and lattices (*see* co-root)
 of observables 232, 240, 247–248
 orbit 81, 219–220
 radical of 60
 reductive 60, 197, 324
 self-dual 324
 semi-simple 60–61, 197, 226
 simple 60, 64, 196
 classification of finite-dimensional 61–63
 presentation 62
 structure 53, 68–71
 simply laced 169

- solvable 60–61, 68, 83–84
of vector fields 36, 55, 178, 184
- Lie derivative 36, 55, 217
- Lie group
classification 59
compact 60, 206
complexification 300
characters at elements of finite order 101, 288, 368
representations 82–85
definition 55–56
and Lie algebras 57–59, 64, 77–78, 206, 300
and special functions 159
and physics 231, 234–235, 254–255, 267–269, 271, 286
representations 44, 82, 85, 184
structure 59
- line bundle
definition 38
determinant 185, 300
sections 38, 84, 178
- link
definition 41
invariant 42, 94, 167, 265, 385
(see also Jones polynomial)
mirror image 42, 44, 400
(see also knot)
- Liouville's Theorem 116
- Littlewood–Richardson rule 370
- local conformal net 391
- local coordinates 34–35, 37, 185, 229, 322
- localisation functor 185
- locality (in physics) 237–238, 254, 318–320, 322, 330
- loop (in quantum field theory) 278–279, 283
- loop algebra 178, 189–190, 192, 205–207, 215
(see also affine algebra, nontwisted/twisted – construction)
- loop group 189, 194, 206–207, 312
- loop space \mathcal{LM} 299, 352
- Lorentz group $\mathrm{SO}_{3,1}^+(\mathbb{R})$ 56, 108, 234–235, 254–255
- Maass form 156, 166
- Magic Square (see Freudenthal's Magic Square)
- Magic Triangle (see Cvitanović's Magic Triangle)
- manifold
ALE 204
conformal 34
definition 33–34
invariants 306
Riemannian 34, 37–38, 229, 236, 351, 353
smooth vs topological structure 34, 56, 296–297
symplectic 85, 231, 232, 270
- mapping class group $\Gamma_{g,n}$
and conformal field theory 185–186, 288, 291, 302, 392
(see also chiral block)
definition 120–122
enhanced $\widehat{\Gamma}_{g,n}$ 125, 299, 420
and Monstrous Moonshine 288, 291, 427
projective representations of 125, 186, 203, 217, 288, 291, 302, 392
(see also braid group \mathcal{B}_3 ; braid group \mathcal{B}_n)
- Markov move 43, 385, 388
- Mathieu groups 19, 402, 404, 409, 422, 426, 432
and Leech lattice 20, 422
- Maxi-Moonshine 145, 292, 294, 424–426, 428
- McKay correspondence 171, 204–205, 374
(see also A-D-E)
- McKay equation 3–4, 135, 402
- McKay–Thompson series $T_g(\tau)$ 5, 78, 95, 145, 218, 220, 407–408
and the Leech lattice 409
linear dependencies 337, 407–408
and modular equations 418
and replicability 409–411, 415
- meromorphic function 114, 116–117
- metaplectic group $\mathrm{Mp}_2(\mathbb{R})$ 163–164, 165–166, 167
- minimal model (CFT) 288
- minimal polynomial 96, 102
- Mini-Moonshine 422–424
- Mirror Moonshine 432–433
- mirror symmetry 204–205, 225, 351, 395, 432–433
- Möbius transformations 107–108, 126, 155, 186, 290, 309, 327
- mode 308, 314, 317, 330, 334
- modular data 183, 196, 288, 342–343, 359–361, 367–371, 376, 382, 390
and Galois 358, 360, 371, 397–398, 400
- modular equation 417–419, 422
- modular fiction 411–412, 418–419, 422
- modular form 196
definition 127–128, 155
of fractional weight 127, 129, 132, 165, 421
for $\mathrm{SL}_2(\mathbb{Z})$ 343, 352
Siegel 152–153, 155
vector-valued 134–135, 343
(see also automorphic form; Borcherds' lift)
- modular function
definition 2, 127
for $\mathrm{SL}_2(\mathbb{Z})$ 2–3
vector-valued 196, 199
(see also Hauptmodul)
- modular functor (Segal) 302–303, 307, 310, 322, 349–350, 376, 386
- modular group $\mathrm{SL}_2(\mathbb{Z})$ (see $\mathrm{SL}_2(\mathbb{Z})$)
- modular invariant 361–362, 373–374, 377, 383, 390
(see also partition function)

- Modular Moonshine 428–430
 modular representation 25–26, 28, 29, 428–429
 modular tower 158, 292, 397
 module
 completely reducible 22, 67
 direct sum 66
 indecomposable 23
 irreducible 22, 67
 simple (*see* irreducible *above*)
 submodule 67
 module for group
 contragredient 22, 25
 definition 20, 82
 direct sum 21–24, 84
 dual (*see* contragredient *above*)
 and representation 20
 tensor product 22, 24–25, 28
 unitary 22–23, 82–83
 from VOA 329, 331
 (*see also* character; module)
 module for Lie algebra
 for abelian \mathfrak{g} 86
 admissible (*see* level, fractional)
 contragredient 66
 definition 66
 derived 82–83, 86, 156, 161, 207
 direct sum 66–67, 76
 dual (*see* contragredient *above*)
 highest-weight 67, 74, 192–193
 integrable 194–196, 205, 368
 vs Lie group module 66, 77, 82–84, 207
 and representation 66
 for solvable \mathfrak{g} 68
 tensor product 66
 twisted 218–220
 unitary 66–67, 86, 194
 Verma 67, 70, 74–77, 192–193
 from VOA 324
 (*see also* affine algebra; character; module)
 module for vertex operator algebra
 in CFT 286, 289, 309
 characters (*see* characters; graded dimension;
 vertex operator algebra)
 contragredient 332
 definition 309, 330
 dual (*see* contragredient *above*)
 graded dimension (*see* graded dimension; vertex
 operator algebra)
 lowest-weight space M_h 332, 334–335
 twisted 220, 293–295, 345–348, 425
 unitary 332
 moduli space $\overline{\mathfrak{M}}_{g,n}$
 in CFT 124–125, 185–186, 281, 283, 285,
 290–292
 definition 119–121
 Deligne–Mumford compactification (*see*
 Deligne–Mumford compactification)
 enhanced $\overline{\mathfrak{M}}_{g,n}$ 124–125, 185–186, 281, 287,
 291, 299
 in string theory 124, 278–280
 and Virasoro, Witt 84, 185–186, 281, 287, 292,
 301, 318, 348, 421
 momentum 231, 240, 242–243, 254, 256–257
 canonical (*see* generalised *above*)
 generalised 232, 238, 255
 monodromy representation 200–203, 204–205
 in CFT 11, 186, 290–291
 and KZ equation (*see* Knizhnik–Zamolodchikov
 equation)
 and modularity 200, 202–204, 291
 in Moonshine 11, 150
 Monster \mathbb{M}
 as 6–transposition group 11, 166, 426–427, 431
 as $\text{Aut}(\Sigma)$ 121
 centralisers in 403–404, 414, 423, 425, 429–431
 character table 403, 405, 407
 conjugacy classes 5, 403, 405
 history 19–20, 403
 representations 4–5, 23–24, 177, 405
 size (order) 4
 and other sporadics 20, 403, 430
 and V^\natural 9, 414
 Monster Lie algebra \mathfrak{m} 214
 construction 415–416
 denominator identity 222, 223, 225, 415–416
 and Monster 415–416
 Monstrous Moonshine
 conjectures 5, 8, 407
 and physics 9–10, 303, 433–434
 Moonshine module V^\natural 328
 automorphism group 406, 414, 426
 construction 294, 347, 414
 graded dimension $J(\tau)$ 294, 343, 407
 and Griess algebra 325, 414
 invariant bilinear form 415
 twisted modules 220, 425, 428, 430, 434
 moonshine-type 138–140, 407, 411, 423, 425
 morphism (*see* arrow)
 Mostow Rigidity Theorem 110, 122
 M-theory 212, 280, 375
 multiplier (for modular forms) 127, 131–132, 134,
 165, 343
 mutually local 272–273, 316, 322

 Nahm’s Conjecture 372
 NIM-rep 361, 374–375, 383, 390–391
 Noether’s Theorem 231–232, 240, 256, 259, 268,
 284
 No-Ghost Theorem 416
 non-commutative geometry 117, 265, 271, 380

- non-orientable surface 111
 normal-ordering 181, 198–199, 258, 317, 335–336, 371
 Norton series $\mathcal{N}_{(g,h)}(\tau)$ 145, 425, 428, 432
 Norton's Conjecture (*see* Maxi-Moonshine)
n-point function (*see* correlation function)
n-torus 33, 84, 123, 205, 215, 250, 393–394
 null vector 67, 76, 216, 290, 308, 327, 330
- objects in category 87
 octonions 53, 56, 61
 operator product expansion (OPE) 266, 283–285, 301, 317, 326–327, 357
 orbifold 204, 292
 and CFT 293–295
 holomorphic 292–294, 347, 382, 384, 392
 and Maxi-moonshine 292, 294, 295, 425–426
 permutation 295, 426
 and strings 292–293
 and VOAs 328, 345–348
 (*see also* module from VOA \mathcal{V}_τ , twisted)
 orbit method 85, 137, 184–185, 207
 orbital integral 137
 order $\|G\|$ of group 15
 oscillator algebra \hat{u}_1 197–198, 223, 325–326, 377
- p*-adic numbers 58, 157–158, 292, 396, 399–400
 pair-of-pants 125, 279, 297, 301, 307, 322–323, 350, 363
 paragraph 389–390
 pariahs 424
 particle 154, 237, 254, 256–259, 273
 partition function 260, 283, 287, 289, 301–302, 314, 353, 361, 373
 (*see also* modular invariant)
 path integral 250–251, 266, 279–280, 300
 (*see also* Feynman path formulation)
 pentagon axiom 89–90, 398–399
 Perron–Frobenius theory 172–173, 175, 357, 374, 387, 389, 391
 Peter–Weyl Theorem 82, 84–85
 phase space 119, 232, 242, 248
 Picard's Theorem 116
 Planck's constant \hbar 242
 Poincaré group 17, 56, 179, 234, 247, 250, 254, 267–268, 272
 Poincaré–Birkhoff–Witt Theorem 74–75, 182
 Pointrjagin duality 136
 Poisson bracket 231, 232, 238, 247, 255
 Poisson summation formula 131, 135–138, 140
 and theta function modularity 8, 104, 131, 134, 137–138, 143, 152
 positive energy representations 182, 207, 300
 potential *V* 227–229, 242
p-regular element 428
- primaries (sectors) 286–287, 356–357, 360, 386
 (*see also* state, primary)
 principal gradation 193–194
 profinite completion \hat{G} 396–400
 projective geometry 38–39, 406
 projective limit 157–158, 271, 291–292, 392, 395–396
 projective *n*-space $\mathbb{P}^n(\mathbb{R})$, $\mathbb{P}^n(\mathbb{C})$ 39, 116–117, 353, 370, 393, 431–432
 projective representation 83, 176–179, 186, 207, 218, 345
 and central extensions (*see central extensions*)
 and CFT 125, 186, 285, 291, 295, 296, 299–302
 projectively equivalent 177
 and quantum theories 177, 242, 247, 254, 272
 and two-cocycle 177–178
- quadratic Casimir 86, 156, 199, 201, 288, 327
 quantisation 247–248, 255, 259
 (*see also* geometric quantisation)
 quantum cohomology (*see* Gromov–Witten invariants)
 quantum dimension 101, 337, 357, 381
 quantum double 357, 380, 382, 390
 quantum electrodynamics (QED) 255, 261, 262, 268–269
 quantum field 117, 237, 253–254, 272, 282–283, 312–313, 322
 quantum field theory 117, 226, 252–253
 axiomatisations 271–275, 390
 mathematical difficulties 167, 257–258, 262–264, 265–266, 270–271
 nonperturbative effects/calculations 262, 265, 280
 nonrenormalisable 263–264, 277
 and number theory 154, 264–265, 395, 400–401
 particles (quanta) 256–259, 273
 perturbation 205, 259–262, 279
 renormalisable 253
 quantum group 75, 125, 202, 378–381, 386
 at root of unity 381, 386
 quantum mechanics
 Feynman's formalism 250–251
 Heisenberg's formalism 247–249
 identical subsystems 249–250
 measurement problem 243–246
 perturbation 251–252
 probability 241, 242
 Schrödinger's formalism 241–242, 246–247
 quantum Schubert calculus 370
 quasi-periodic 143, 151, 159
 quasi-primary (*see* state, quasi-primary)
 quasi-symmetric homeomorphism 185
 quaternions 17, 24, 53, 56, 61, 64, 351
 quilt 427

- Racah coefficients (or $6j$ -symbols) 365
 Racah–Speiser formula 369
 Ramanujan τ -function 142, 221
 rational conformal field theory (*see* conformal field theory, rational)
 ray representation (*see* projective representation)
 regular representation (*see* group algebra; $L^2(G)$)
 regularisation
 and Lie theory 198–199, 270, 371
 and number theory 118, 133, 270
 in quantum field theory 133, 263–264, 270–271
 zeta-function 133, 140, 199
 Reidemeister moves 42, 383, 391
 relativity
 general 226, 233, 236, 239, 263, 267–268, 269, 277
 special 233–236, 237, 238, 246, 247, 250, 252, 254, 272
 renormalisation 260, 262–265, 270–271
 replicable functions 410–412, 415–416, 422, 426
 and the power map g^n 409
 representation (*see* module)
 ρ -shift (Lie algebras) 217, 371
 ribbon 93
 (*see also* category, **Ribbon**)
 Riemann sphere 2, 39, 138, 285
 Riemann surface 87, 114–117, 216–217
 in CFT 264, 277, 281–282, 286, 298
 and conformal structure 114, 281, 290
 Riemann zeta function $\zeta(s)$ 127–128, 133, 140–142, 154, 168–169, 199
 Riemann–Hilbert problem 203
 R -matrix universal (*see* universal R -matrix)
 root 68, 73
 highest 193
 imaginary 68, 191, 193, 211, 212, 214, 224–225, 415–416
 lattice (*see* lattice, root)
 positive 70, 73–75, 77, 193, 220, 415–416
 real 191, 193, 211, 214, 224, 415–416
 simple 69–70, 72, 79, 187, 190, 193, 214, 415–416
 space 69–70, 415–416
 space decomposition 69, 86, 191, 210, 213, 415–416
 system 69–70, 72
 scale invariance 230, 284–285, 297
 scattering matrix (*see* S -matrix)
 Schur multiplier 177–179
 Schur polynomial 72
 Schur’s Lemma 23, 67, 80, 161, 192, 216, 331
 Schrödinger’s equation 242, 243, 245, 246, 247, 253, 275–276
 Schwartz space $\mathcal{S}(\mathbb{R}^n)$ 45–49, 131, 136, 162, 241, 253
 Schwarzian derivative 340
 sector
 superselection 242, 296
 twisted 293–295, 392
 Segal’s axioms (CFT) 298–303, 305, 310, 348–350
 Selberg trace formula 137, 154
 self-adjoint operator 47, 49, 52, 242, 247, 275, 354–355
 semi-direct product 16–17, 18, 28–29, 56, 85, 195
 semi-direct sum 53, 59–60, 406
 sewing 293, 299–302, 306–307, 310, 339, 341, 363–364, 397
 sheaf 34–35, 351–352
 Siegel upper half-space \mathbb{H}_g 122–123, 151–152, 155
 simple factor (in geometry) 393–394
 simple-current 357–358, 360, 362, 367, 369, 373, 391
 simply connected 40, 43, 83, 115, 184, 200, 202, 206–207
 singular vector (*see* null vector)
 singularity
 blowing up 117
 minimal resolution 171, 204–205, 402
 quotient (orbifold, conical) 116, 120–121, 170, 204, 292, 374, 402
 resolution 171
 simple 170–171, 204
 (*see also* A-D-E; McKay correspondence)
 $SL_2(\mathbb{R})$ 86, 107, 109–110, 125, 160
 and Lorentz group 108, 154
 and modular forms 154–157, 223
 representations of 86, 155–156
 universal cover 11, 164–165, 167, 184
 $SL_2(\mathbb{Z})$ 109–110, 116, 126, 152
 in CFT 205, 288, 291, 293
 representations of 177, 293, 342, 359
 (*see also* modular data)
 and tori 10, 120–121, 130, 280
 (*see also* braid group B_3 ; modular form; modular function)
 S -matrix 154, 252, 259, 265, 281
 space-time 34, 229, 265, 277, 280–281, 292, 297, 301
 Minkowski 29, 56, 233–234, 236, 239, 268, 274, 281–282
 Spectral Theorem 48–49, 242–243, 273
 speed c of light 233–235, 237, 255, 282
 sphere S^n 40, 42, 53, 55–56
 sphere-packing 31–32
 spin 83, 250, 254–255, 296, 352
 spinor 83
 sporadic group (*see* finite simple group)
 squirrel 12, 59, 329, 435
 Standard Model 255–256, 262–264, 266, 269, 273, 277–278, 280
 star-triangle relation (*see* Yang–Baxter equation)

- state
 BPS 205, 214
 incoming 258–259, 266, 273, 278–279, 283, 300–301
 outgoing 258–259, 266, 273, 278–279, 300–301
 primary 285–286, 287, 323, 325, 340
 quasi-primary 285, 366
 state space \mathcal{H} 249, 253–254, 272, 275, 286, 298–299, 322, 361
 state-field correspondence 283, 318
 statistical mechanics 266, 281
 Stone–von Neumann Theorem 161, 163, 265
 stress–energy tensor 239–240, 284–285, 288, 292, 301, 322
 string theory 277–280
 and CFT 9, 252, 276–277, 279, 281, 289, 300
 and lattices 10, 280, 293–294, 360
 (see also vertex operator algebra, lattice)
 and Lie groups (see Wess–Zumino–Witten model)
 and modular forms 9–10, 166, 264, 279–280, 408
 and moduli space (see moduli space in string theory)
 and Monstrous Moonshine 280, 433–434
 perturbative 9, 252, 264, 276, 278–279, 433
 subfactor
 basic construction 387–388
 and braids 27, 125, 167, 202, 288, 388
 and CFT 44, 374, 386, 388–391
 definition 49, 386
 analogy with Galois 386, 389
 and knots 49, 386–388
 and orbifolds 391, 426
 subgroup
 commutator $[GG]$ 29
 index (see index, subgroup)
 normal 14–17, 24, 59
 Sugawara construction 199, 217, 324, 326
 superposition 243–245
 supersymmetry (see conformal field theory, super; vertex operator superalgebra)
 surface 33
 enhanced 124–125, 185, 281, 283, 287, 291, 300, 349
 K3 (see K3 surface)
 with nodes 124, 280, 291, 292
 (see also cusp; Deligne–Mumford compactification)
 Riemann (see Riemann surface)
 stable (see surface with nodes)
 symmetric group \mathcal{S}_n 17, 20–24, 93, 100, 187–188, 250, 296
 as Weyl group 71, 78, 220
 (see also braid groups)
 Tamanoi’s invariant 351, 353
 tangent
 bundle TM 35, 38, 53, 56
 space $T_p(M)$ 35–38, 56–59
 vector 35–38
 Taniyama–Shimura conjecture 130, 142
 Tannaka–Krein duality 90, 136, 328
 Teichmüller space $\mathfrak{T}_{g,n}$ 120–122, 291
 universal 185
 theta function
 Jacobi 118–119, 131, 142–143, 160, 203, 223–224
 modularity 8, 104, 131, 138, 143, 147–148, 163–164, 280
 Siegel’s 123, 151
 theta series, lattice 7, 134–135, 138, 140, 143, 195–196, 280, 294
 Thompson trick 4–5, 24, 80, 307, 346, 424, 425–426
 topological field theory 100, 303, 305–307, 363, 388, 421
 and conformal field theory 287, 305, 307
 topological span 46, 241, 253
 toroidal algebra 215–216
 torus $S^1 \times S^1$
 and CFT 287, 292–293, 301–302
 conformal structures on 44, 120, 301, 341
 diffeomorphisms 44
 and elliptic curves 110, 112, 118, 123–124
 fundamental group 40, 113
 (see also elliptic curve; n -torus; $SL_2(\mathbb{Z})$)
 tower 157, 158, 291, 397
 trace
 in CFT 241, 293, 300
 as character 5, 23, 25, 77, 183
 and determinant 58
 and sewing 341
 in von Neumann algebras 50–52
 trefoil 41–42, 164–166, 383–384, 391
 triangle axiom 89–90, 398–399
 triangular decomposition (Lie algebra) 70, 182, 191–192, 210, 213
 Turaev–Viro theory 385, 389
 twenty-four 168–169, 198, 394, 402, 414, 416, 419, 425
 (see also $c/24$)
 twining character (see character, twisted)
 uniformisation 3, 115–116
 unitary dual \widehat{G} 82–86, 136–137
 unitary operator 22, 47–49, 247, 254
 unitary representation (see Lie algebra; module for group; VOA)
 universal cover 115–116, 200–201
 group \widetilde{G} 59, 83, 165, 178, 234, 254, 291
 universal enveloping algebra $U(\mathfrak{g})$ 74–75, 156–157, 182, 198–199, 327, 334, 378–380
 universal R -matrix 380

- unknot 41
- upper half-plane \mathbb{H}^2 105, 110, 120, 127, 154
(*see also* hyperbolic plane)
- V^{\natural} (*see* Moonshine module)
- vacuum (sector) 194, 302, 325, 342, 356, 360–361, 368, 382, 390
- vacuum $|0\rangle$ (state) 248, 253, 256–257, 259, 272–273, 280, 301, 318–319, 322–323
- vacuum-to-vacuum expectation value (*see* correlation function)
- Vandermonde matrix 221
- variety 94, 351, 393–395, 396
- $\text{Vect}(M)$ 36, 55, 59, 178, 184
- $\text{Vect}(S^1)$ (*see* Witt algebra)
- vector bundle
 - base of 38
 - connection on 38
 - definition 38
 - fibre of 38
 - G -equivariant 382
 - section of 38, 134, 291
- vector field 35–36, 53, 55, 57, 178, 184, 188
- Verlinde dimension 287
- Verlinde's formula 217, 287, 343, 357, 359, 366, 376, 392
- Verma module $M(\lambda)$ 67, 70, 74–77, 182, 185, 192–194, 211, 214, 217
- vertex algebra 318–319, 351
- vertex group 350
- vertex operator 279, 285, 311–312, 318, 322–323, 330, 363–364, 413
- vertex operator algebra (VOA) 311, 330, 339, 350
 - affine algebra 215, 326–327, 329, 402
(*see also* integrable affine *below*)
 - in conformal field theory 285–286, 289, 293, 301, 309–310
 - definition 8, 311, 318–319, 390
 - geometric 348–350, 391
 - Heisenberg 326, 329, 333, 377
 - holomorphic 333, 347, 348, 382
 - integrable affine 194, 286–287, 327–330, 333, 335–339, 348, 376
 - lattice 327–329, 332–333, 347, 348
 - and lattice analogy 328–329, 347, 414–415
 - and Lie algebras 8, 324–325, 333, 336, 338, 351
 - near- 215, 318, 328–329, 415
 - rational 286, 330–331, 375–376
 - simple 322, 331, 375
 - weakly rational 324, 331–333, 336–337
- vertex operator superalgebra 322, 328, 351–353, 406, 414, 423–424, 429–431
- Virasoro algebra \mathfrak{Vir} 181–186
 - and affine algebras 198–199, 270
 - and CFT 185–186, 281, 284–288, 299, 301
 - definition 140, 181
 - higher-dimension generalisation \mathfrak{V}_τ 215
 - higher-genus generalisation $\widehat{\mathcal{L}}_{\Sigma, \rho}$ 217
 - and KZ equation (*see* Knizhnik–Zamolodchikov equation)
 - and modularity 183, 318
 - modules 181–184, 323
 - and moduli spaces (*see* moduli space)
 - and VOAs 182, 318, 323, 331
 - and coordinate changes 339–341, 350
(*see also* $\text{Diff}^+(S^1)$; Witt algebra)
- virtual (in quantum perturbation) 252, 260–262, 277
- Vogel's Universal Lie algebra 71, 73, 94
- von Neumann algebra
 - abelian 50
 - centre of 50
 - definition 49–50
 - and KZ equations 421
 - and quantum field theory 50, 275, 390–391
(*see also* factor; subfactor)
- Ward identities 259, 284, 287, 290
- wave-function 179, 241–243, 245
- Weierstrass function $p(z)$ 118–119, 143
- weight (Lie theory)
 - dominant integral 67, 71, 76
 - Dynkin labels of 71, 76, 211, 254
 - fundamental ω_i 71, 73–76, 192
 - highest 67, 74–76, 182, 368
 - highest weight vector 74, 211, 323
 - integrable 194, 196, 199, 205, 207, 211, 214, 219
 - and roots 73
 - space 73–76, 80–81, 190, 218
 - space decomposition 73–76, 86, 190, 192–194, 199
- weight (modular form) 127, 143, 152, 155
- Weil's Converse Theorem 142, 146–147
- Wess–Zumino–Witten (WZW) model 216, 217, 280, 286–288, 295, 371–373, 378, 381, 434
- Weyl character formula 77–78, 81, 84, 150, 211, 220
- Weyl group
 - affine 188, 190, 194–196, 369, 371, 406
 - definition 70, 79, 211, 214
 - finite 185, 187–188, 195–196, 204, 368
 - and lattices 32, 70, 169
 - and weights 70, 73–74, 78, 195
- Weyl reflection 70
- Weyl vector ρ 77, 195, 368, 371
- Weyl–Kac character formula 196, 211, 221
- Weyl–Kac–Borchers character formula 214, 220
- Weyl's dimension formula 223, 225
- Weyl's unitary trick 83
- Wightman axioms 265, 266, 271–275
 - and VOAs 10, 390
- Wirtinger presentation 164, 384, 391

- Witt algebra \mathfrak{Witt}
and affine algebras 189–190, 197–198, 208, 270
and conformal symmetry 185, 281–282, 285
definition 64, 216
as derivations 187, 189–190, 208
and $\text{Diff}^+(S^1)$ 184, 282
higher-genus generalisation $\mathcal{L}_{\Sigma, P}$ 217
its Lie (semi-)group? 184, 300–301
and Mathieu's classification 189
modules of 180–181
and moduli spaces (*see* moduli space)
and $\text{Vect}(S^1)$ 64, 184
and Virasoro algebra 178, 181, 184, 186, 285
(*see also* $\text{Diff}^+(S^1)$; Virasoro algebra)
- world-sheet 216, 278–280, 281–283, 298, 313,
371
wreath product 406, 411
- Yang–Baxter equation 366, 378, 380, 385, 390
and braids 378
and categories 92
classical 378
- zero-mode $o(u)$ 324, 334–335, 348
 \mathbb{Z} -grading as S^1 action 207
Zhu's algebra $A(\mathcal{V})$ 309–310, 334–336, 377
Zhu's Theorem 9, 104, 342–344, 375, 377, 414,
421

